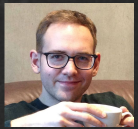
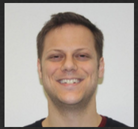
Distributional Reinforcement Learning

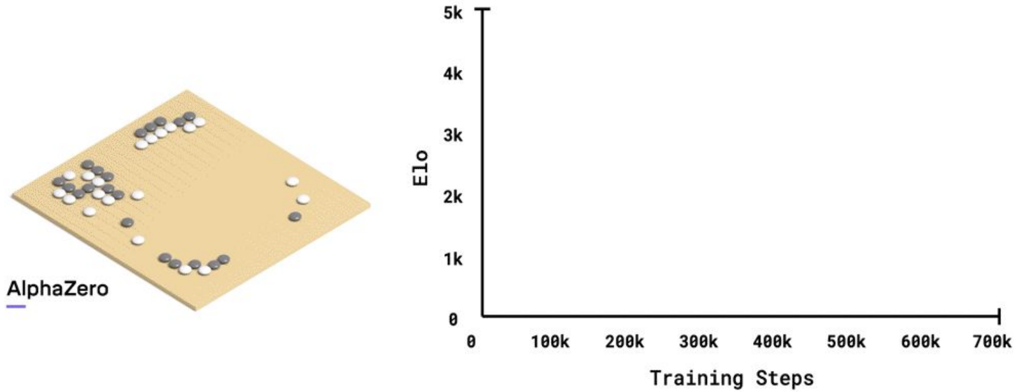
Rémi Munos

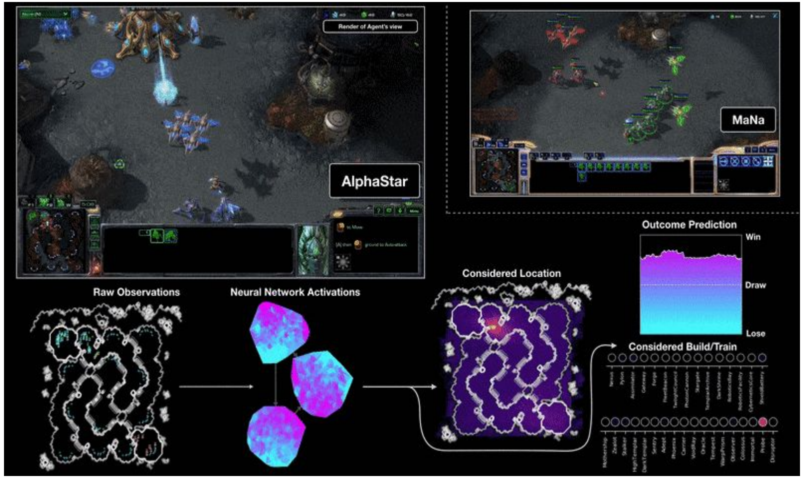
Marc Bellemare, Will Dabney, Georg Ostrovski, Mark Rowland

DeepMindParis

Deep RL at DeepMind



Go chess shogi



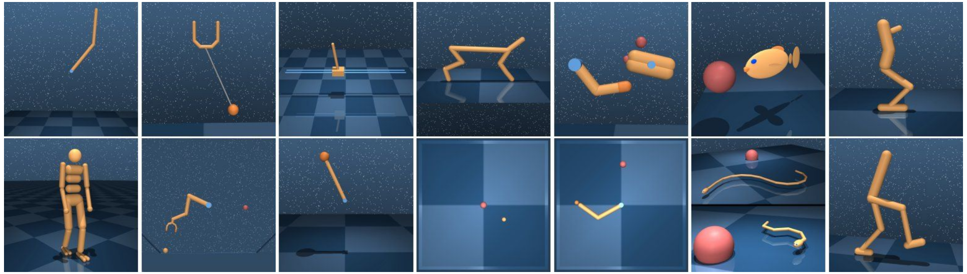
Starcraft

Deep RL at DeepMind

Atari 57 games

DMLab 30

Control suite



One algorithm for all!

Distributional-RL

Outline:

- Brief introduction to (deep) reinforcement learning
- Intro to distributional-RL
    - Theory
    - Representation of distributions
    - Experiments on Atari
- Interactions between RL and deep-learning

Introduction to Reinforcement Learning (RL)

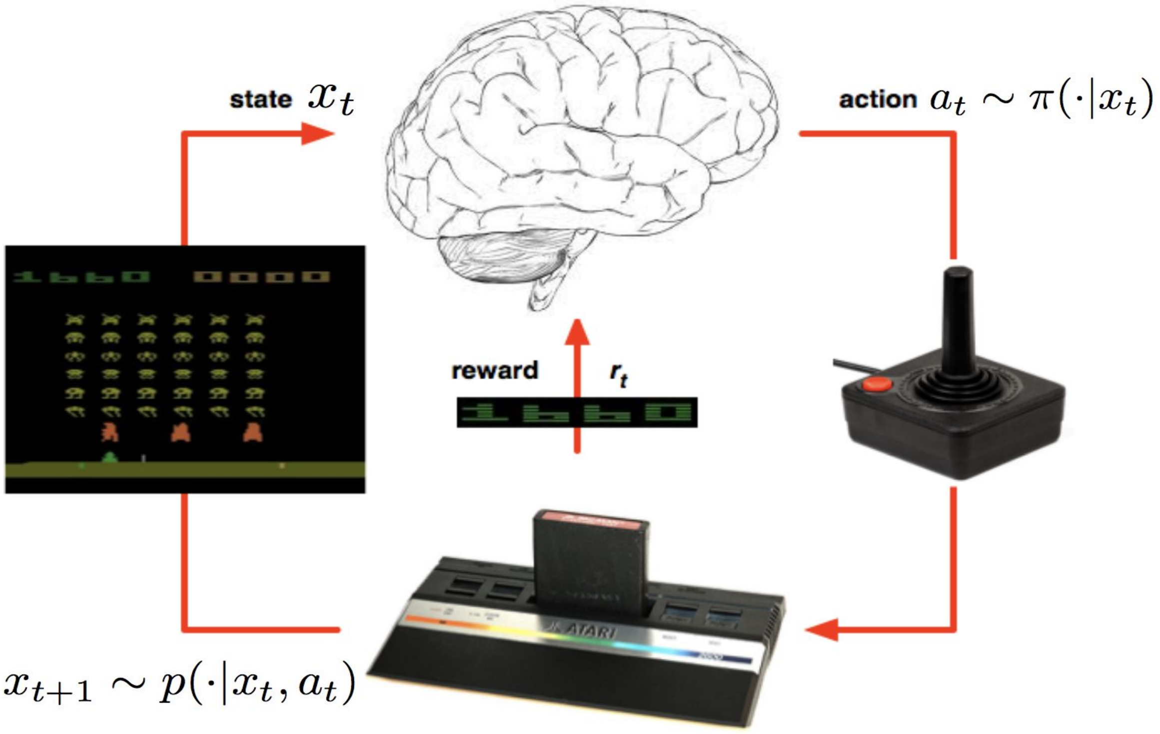
- Learn to make good decisions
- No supervision. Learn from rewards

I learned to ride with RL...

Two approaches:

- Value based ([Bellman, 1957]'s dynamic programming)
- Policy based ([Pontryagin, 1956]'s maximum principle)

The RL agent in its environment



Bellman's dynamic programming

- Define the value function Q^π of a policy $\pi(a|x)$:

$$Q^\pi(x,a) = \mathbb{E}\Big[\sum_{t\geq 0}\gamma^t r_t \Big| x, a, \pi\Big],$$

  and the optimal value function:

$$Q^*(x,a) = \max_\pi Q^\pi(x,a).$$

  (expected sum of future rewards if the agent plays optimally).

- Bellman equations:

$$Q^\pi(x,a) = r(x,a) + \gamma \mathbb{E}_{x'}\Big[\sum_{a'}\pi(a'|x')Q^\pi(x',a')\Big|x,a\Big]$$

$$Q^*(x,a) = r(x,a) + \gamma \mathbb{E}_{x'}\Big[\max_{a'} Q^*(x',a')\Big|x,a\Big]$$

- Optimal policy $\pi^*(x) = \arg\max_a Q^*(x,a)$

Represent Q using a neural network

- Represent value function $Q_w(x, a)$ with a neural net.
- How to train $Q_w(x, a)$? We don't have supervised values. We only know we want

$$Q_w(x, a) \approx r(x, a) + \gamma \mathbb{E}_{x'}\left[\max_{a'} Q_w(x', a') \Big| x, a \right]$$

- After a transition $x_t, a_t \to x_{t+1}$,

$$\text{train } Q_w(x_t, a_t) \text{ to predict } \underbrace{r_t + \gamma \max_a Q_w(x_{t+1}, a)}_{\text{target values}}$$

- Minimize loss $\Big(\underbrace{r_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)}_{\text{temporal difference } \delta_t} \Big)^2$.
- At the end of learning, $\mathbb{E}[\delta_t] = 0$.

Deep Q-Networks (DQN) [Mnih et al. 2013, 2015]

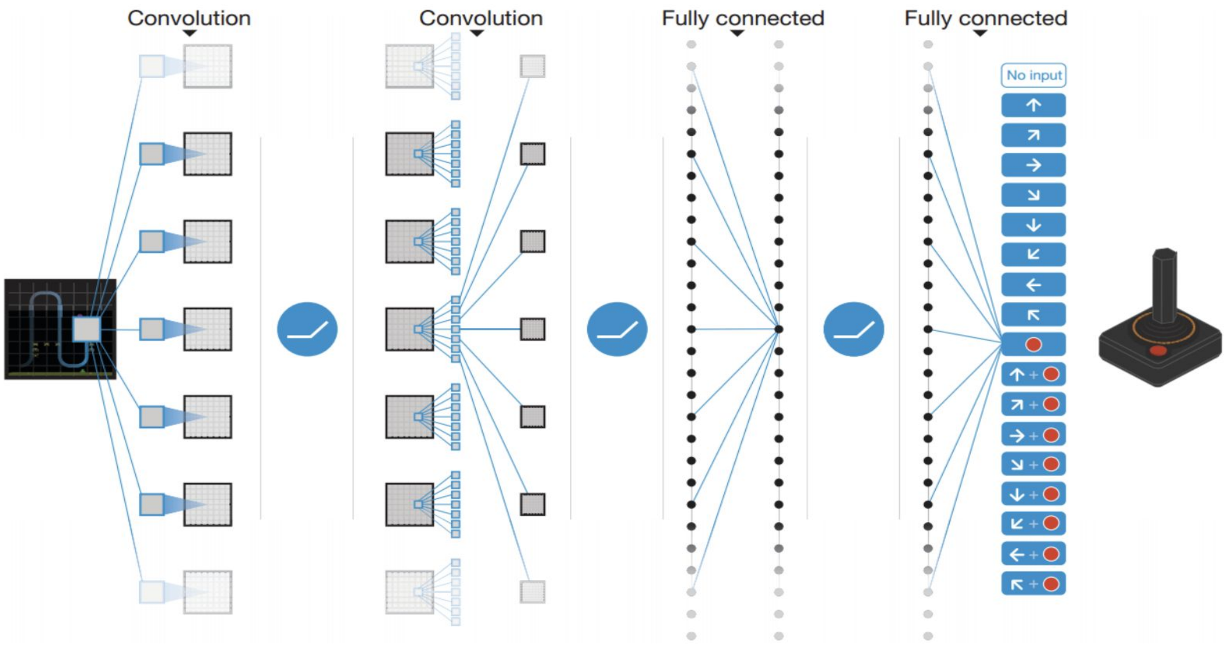
Problems: (1) data is not iid, (2) target values change
Idea: be as close as possible to supervised learning

1. Dissociate acting from learning:
   - Interact with the environments by following behavior policy
   - Store transition samples x_t, a_t, x_{t+1}, r_t into a memory replay
   - Train by replaying iid from memory
2. Use target network fixed for a while

$$\text{loss} \;=\Big(r_t+\gamma\max_a Q_{w_{target}}(x_{t+1},a)-Q_w(x_t,a_t)\Big)^2$$

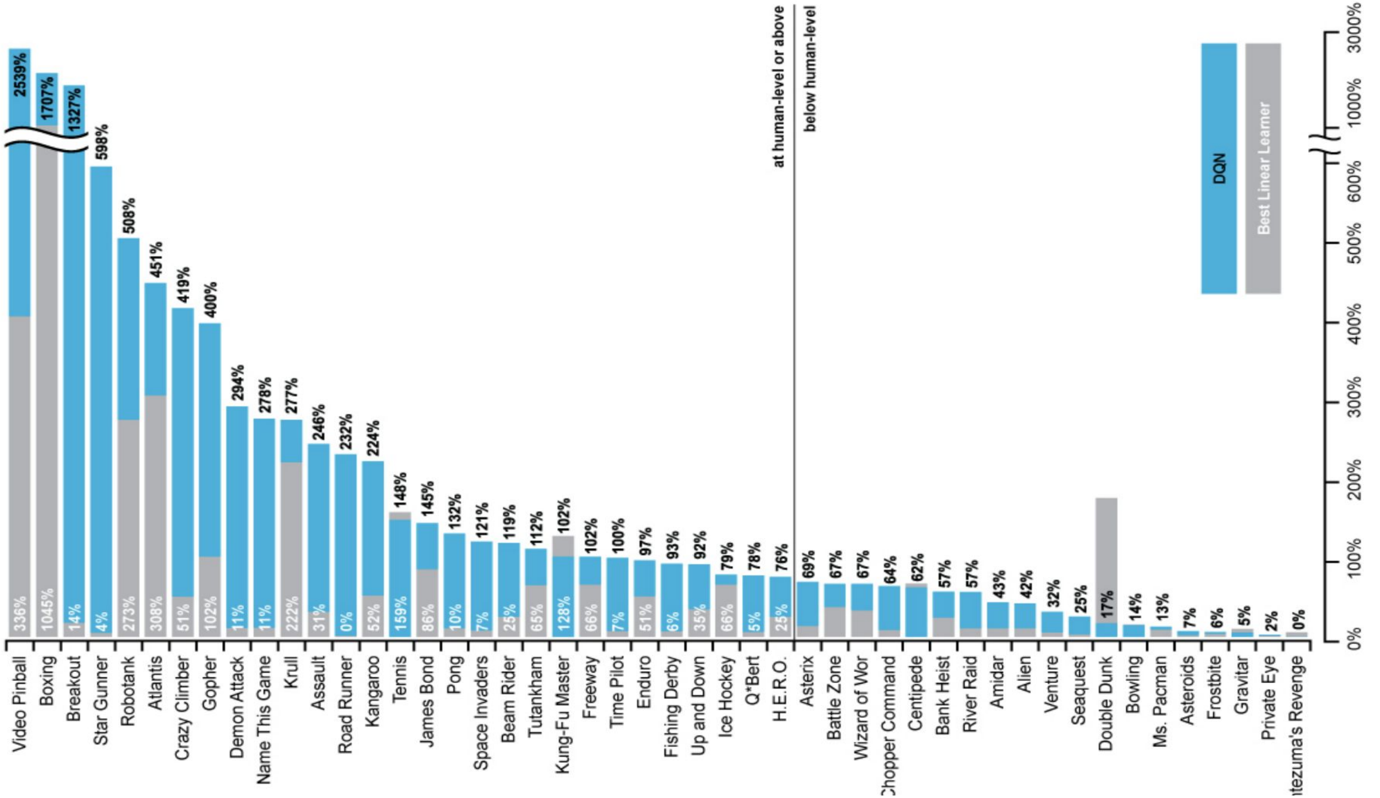
Properties: DQN is off-policy, and uses 1-step bootstrapping.

DQN network



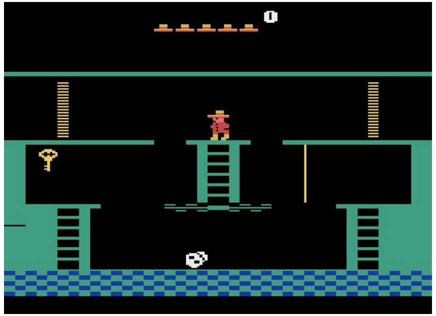
Convolution
Convolution
Fully connected
Fully connected
No input

DQN Results in Atari



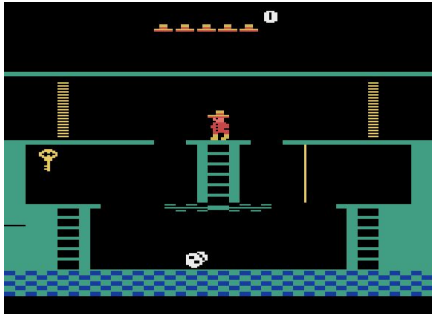
Remi Munos DeepMind

Challenges of deep RL



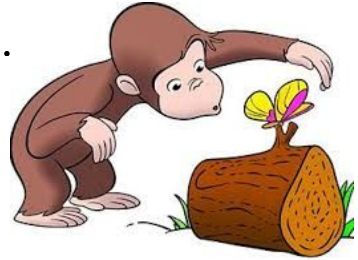
Rewards may be sparse...

Challenges of deep RL

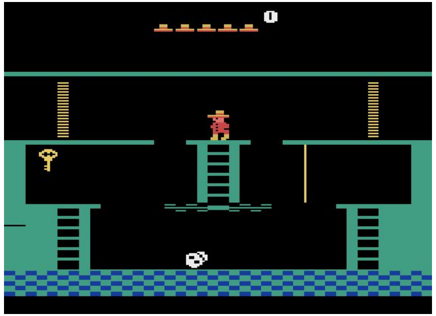


Rewards may be sparse...

Intrinsic motivation, curiosity, learning progress, empowerment, ..



Challenges of deep RL



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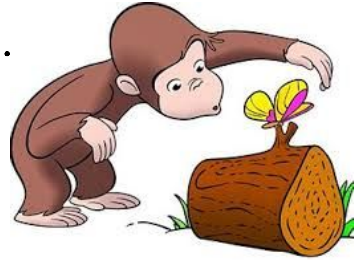
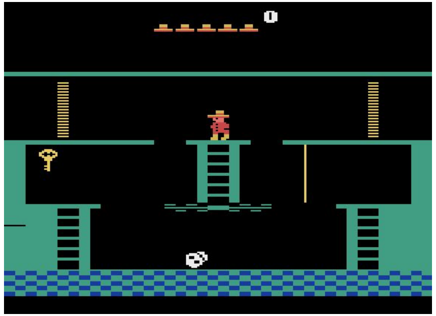


Learn representations in an unsupervised manner

Challenges of deep RL



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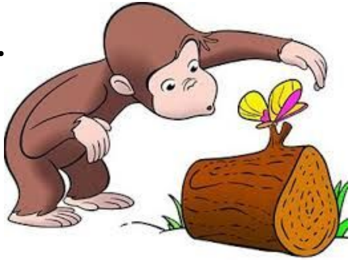
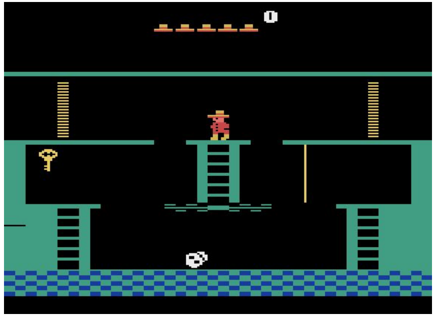


Learn representations in an unsupervised manner

Learn from a teacher

Challenges of deep RL



Rewards may be sparse...

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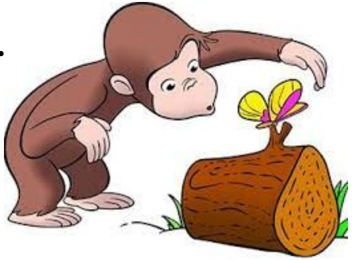


Learn representations in an unsupervised manner

Learn from a teacher

Sample efficiency

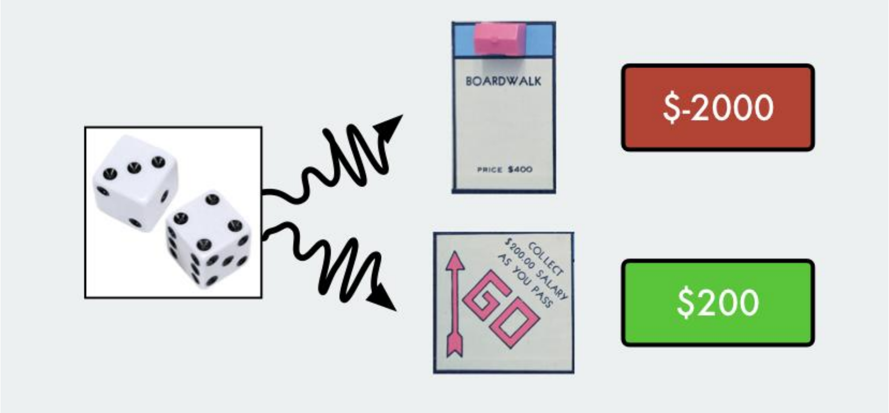
Distributional-RL

- Introduction
- Elements of theory
- Neural net representations
- Experiments on Atari
- Conclusion

Intro to distributional RL

Expected immediate reward

$$\mathbb{E}[R(x)] = \frac{1}{36} \times (-2000) + \frac{35}{36} \times (200) = 138.88$$



Random variable reward:

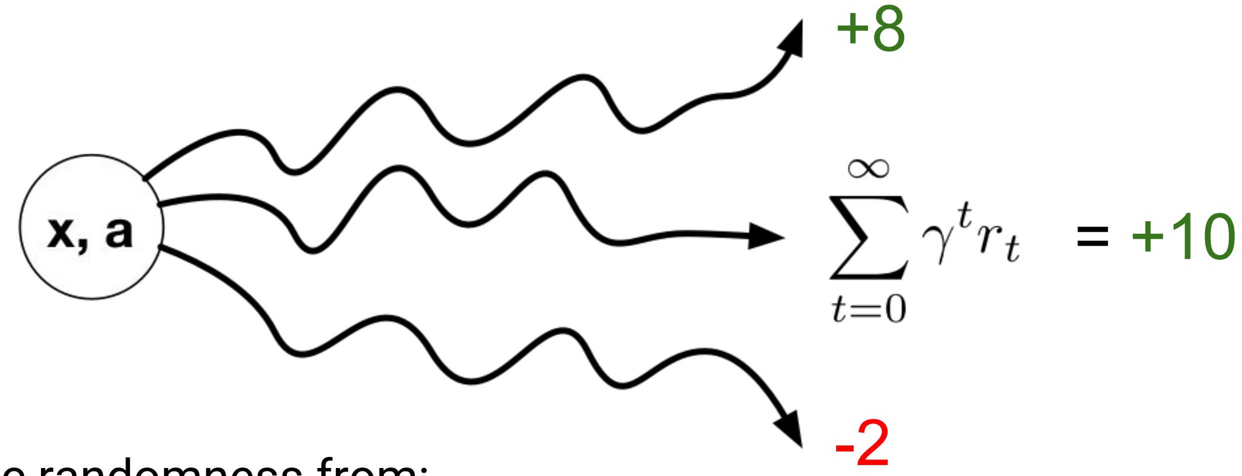
$$R(x) = \begin{cases} -2000 \text{ w.p. } 1/36 \\ 200 \text{ w.p. } 35/36 \end{cases}$$

The return = sum of future discounted rewards

$$R(x_0) + \gamma R(x_1) + \gamma^2 R(x_2) \ldots$$

- Returns are often complex, multimodal
- Modelling the expected return hides this intrinsic randomness
- Model all possible returns!

The r.v. Return $Z^\pi(x,a) = \sum_{t\geq 0} \gamma^t r(x_t, a_t)\big|_{x_0=x, a_0=a, \pi}$



Captures intrinsic randomness from:

- Immediate rewards
- Stochastic dynamics
- Possibly stochastic policy

The expected Return

The value function $Q^\pi(x,a) = \mathbb{E}[Z^\pi(x,a)]$

Satisfies the Bellman equation

$$Q^\pi(x,a) = \mathbb{E}[r(x,a) + \gamma Q^\pi(x',a')]$$

$$\text{where } x' \sim p(\cdot|x,a) \text{ and } a' \sim \pi(\cdot|x')$$

Distributional Bellman equation?

We would like to write a Bellman equation for the distributions:

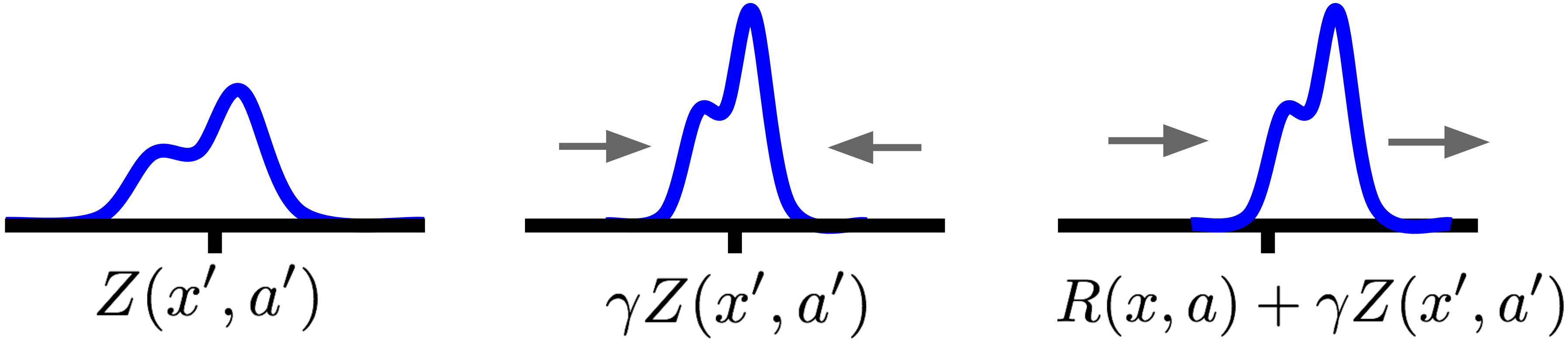
$$Z^{\pi}(x,a) \stackrel{D}{=} R(x,a) + \gamma Z^{\pi}(x',a')$$

$$\text{where } x' \sim p(\cdot|x,a) \text{ and } a' \sim \pi(\cdot|x')$$

Does this equation make sense?

Distributional Bellman operator

$$T^{\pi} Z(x, a) = R(x, a) + \gamma Z(x', a')$$



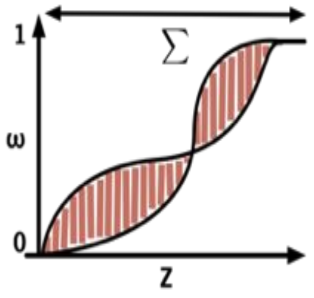
Does there exists a fixed point?

Properties

Theorem [Rowland et al., 2018]

T^π is a contraction in Cramer metric

$$\ell_2(X,Y) = \left(\int_{\mathbb{R}} (F_X(t) - F_Y(t))^2 dt\right)^{1/2}$$

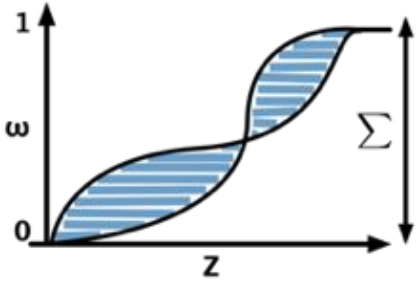


Cramer

Theorem [Bellemare et al., 2017]

T^π is a contraction in Wasserstein metric,

$$w_p(X,Y) = \left(\int_{\mathbb{R}} (F_X^{-1}(t) - F_Y^{-1}(t))^p dt\right)^{1/p}$$



Wasserstein

(but not in KL neither in total variation)
Intuition: the size of the support shrinks.

Distributional dynamic programming

For a given policy π, the distributional Bellman operator

$$T^{\pi} Z(x, a) = R(x, a) + \gamma Z(x', a')$$

Is a contraction mapping, thus has a unique fixed point, which is Z^{π}

And the iterate $Z \leftarrow T^{\pi} Z$ converges to Z^{π}

The control case

Define the distributional Bellman optimality operator

$$TZ(x,a) \stackrel{D}{=} r(x,a) + \gamma Z(x', \pi_Z(x'))$$

$$\text{where } x' \sim p(\cdot|x,a) \text{ and } \pi_Z(x') = \arg\max_{a'} \mathbb{E}[Z(x',a')]$$

Is this operator a contraction mapping?

The control case

Define the distributional Bellman optimality operator

$$TZ(x,a) \stackrel{D}{=} r(x,a) + \gamma Z(x', \pi_Z(x'))$$

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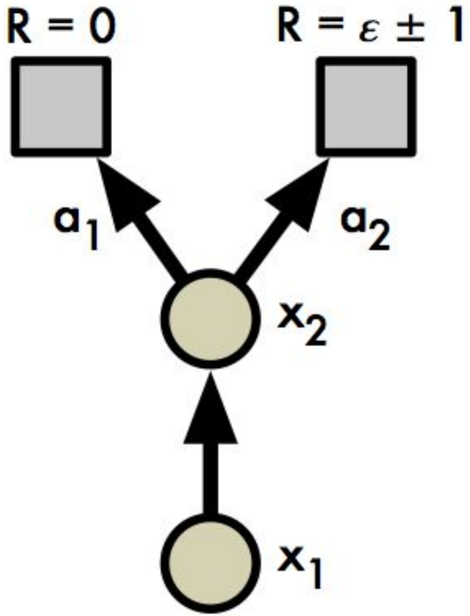
Is this operator a contraction mapping?

No! (it's not even continuous)

The dist. opt. Bellman operator is not smooth



Consider distributions Z_ϵ

If ε > 0 we back up a bimodal distribution

If ε < 0 we back up a Dirac in 0

Thus the map $Z_\epsilon \mapsto TZ_\epsilon$ is not continuous

Distributional Bellman optimality operator

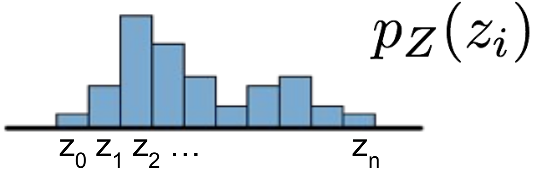
Theorem [Bellemare et al., 2017]

if the optimal policy is unique, then the iterates $Z_{k+1} \leftarrow TZ_k$ converge to Z^{π^*}

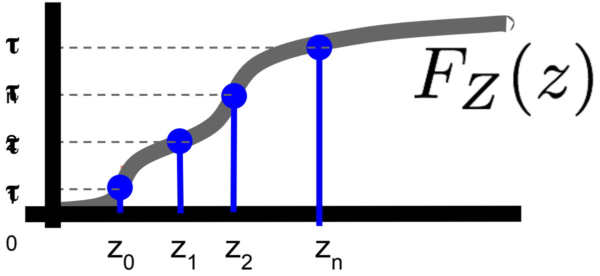
Intuition: The distributional Bellman operator preserves the mean, thus the mean will converge to the optimal policy π^* eventually. If the policy is unique, we revert to iterating T^{π^*}, which is a contraction.

How to represent distributions?

- Categorical



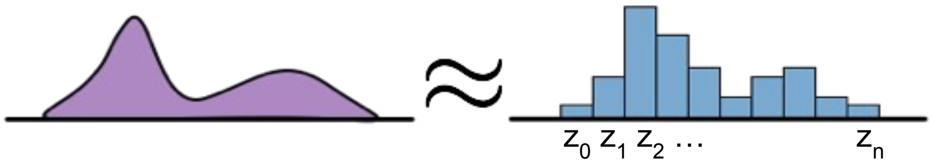
- Inverse CDF for specific quantile levels



- Parametric inverse CDF

$$\tau \mapsto F_Z^{-1}(\tau)$$

Categorical distributions



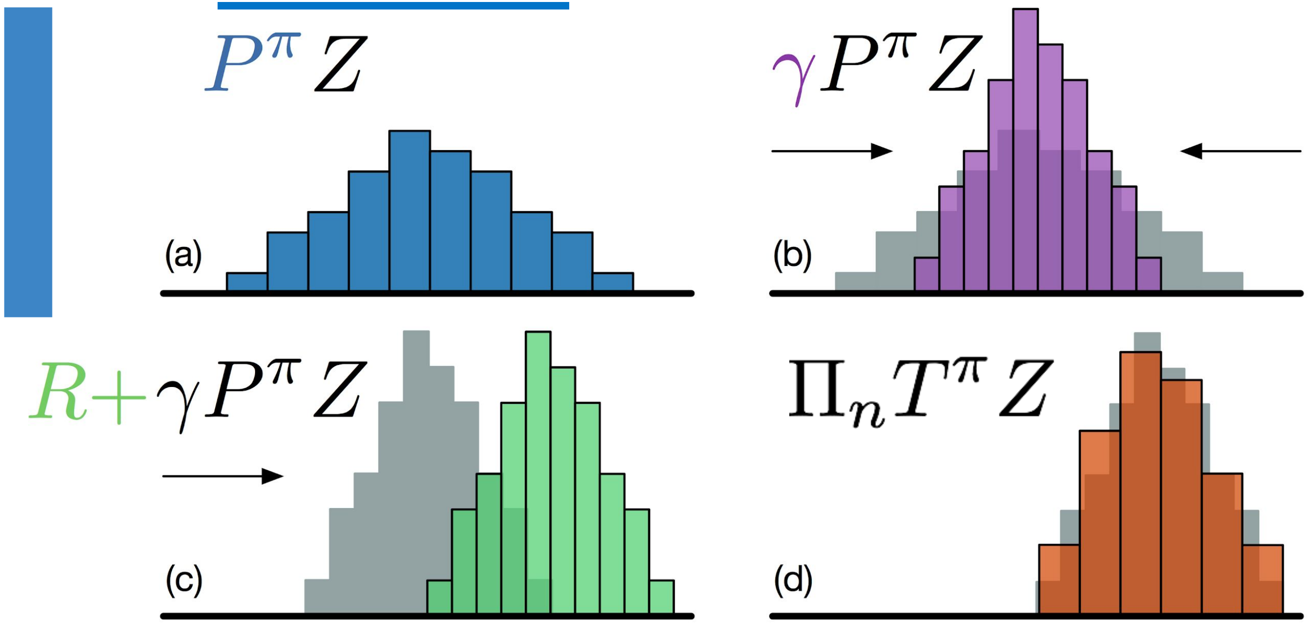
Distributions supported on a finite support $\{z_1, \ldots, z_n\}$

Discrete distribution $\{p_i(x, a)\}_{1 \le i \le n}$

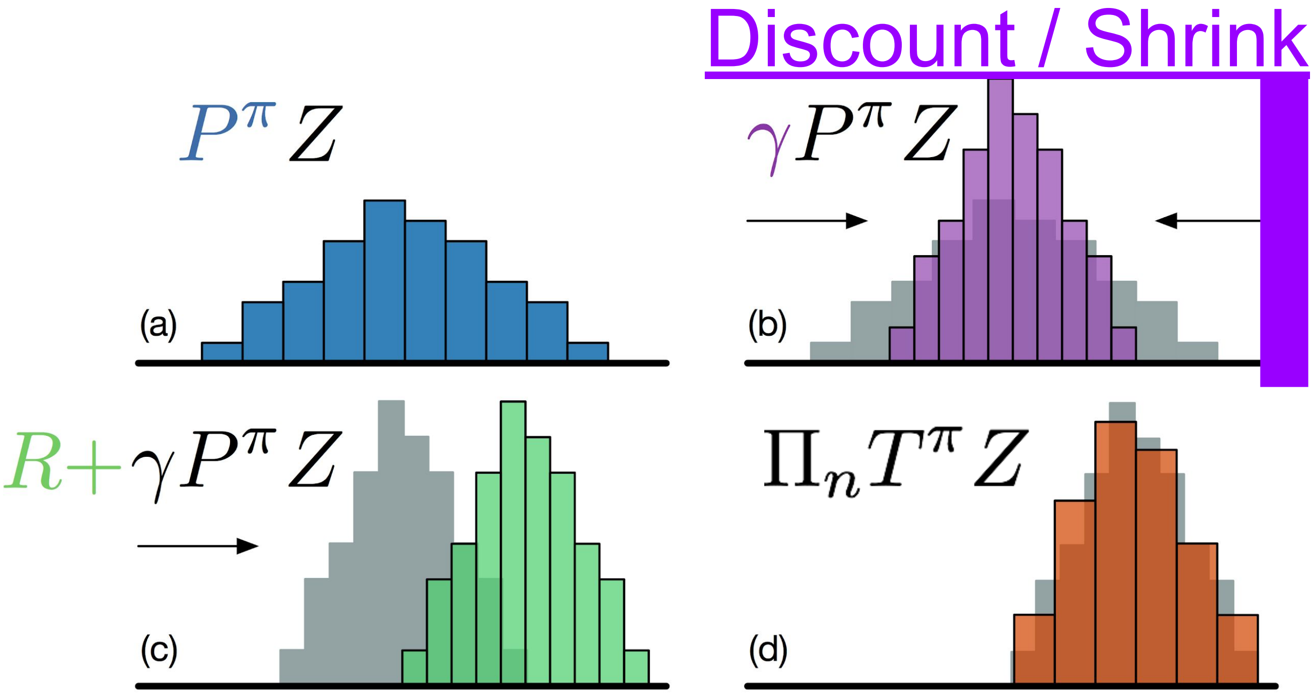
$$Z(x, a) = \sum_i p_i(x, a) \delta_{z_i}$$

Projected Distributional Bellman Update

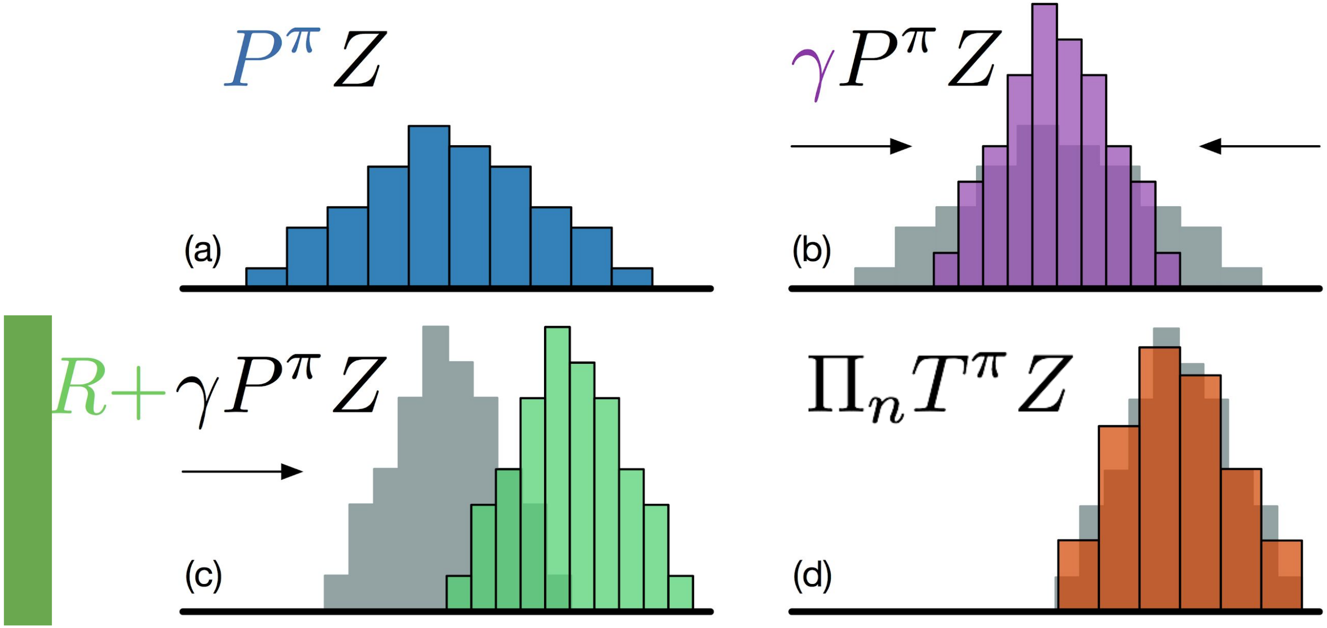
Transition



Projected Distributional Bellman Update

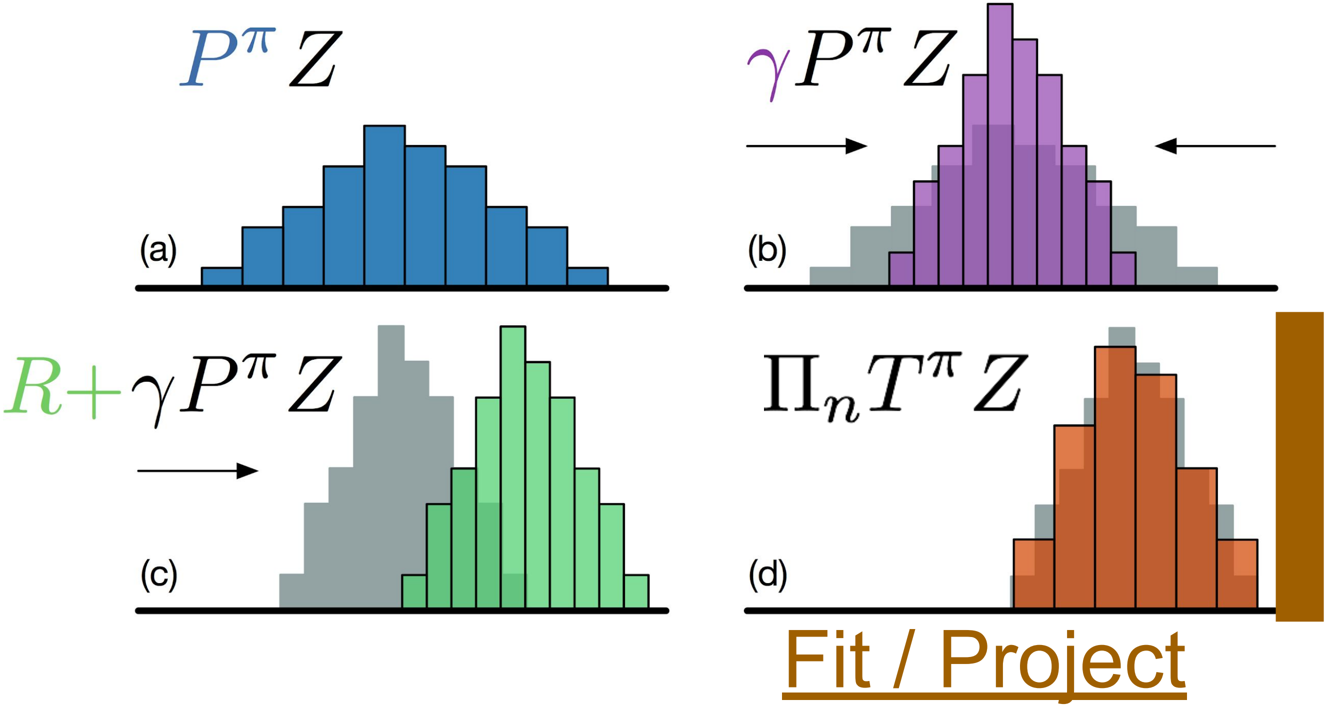


Projected Distributional Bellman Update



Reward / Shift

Projected Distributional Bellman Update



Projected distributional Bellman operator

Let Π_n be the projection onto the support (piecewise linear interpolation)

Theorem: $\boxed{\Pi_n T^\pi \text{ is a contraction (in Cramer distance)}}$

Intuition: Π_n is a non-expansion (in Cramer distance).

Its fixed point Z_n can be computed by value iteration $Z \leftarrow \Pi_n T^\pi Z$

Theorem: $\boxed{\ell_2^2(Z_n, Z^\pi) \leq \frac{1}{(1-\gamma)} \max_{1 \leq i < n} |z_{i+1} - z_i|}$ [Rowland et al., 2018]

Projected distributional Bellman operator

Policy iteration: iterate

- Policy evaluation: $Z_k = \Pi_n T^{\pi_k} Z_k$
- Policy improvement: $\pi_{k+1}(x) = \arg\max_a \mathbb{E}[Z^{\pi_k}(x,a)]$

Theorem: Assume there is a unique optimal policy.
Z_k converges to $Z_n^{\pi^*}$, whose greedy policy is optimal.

Distributional Q-learning

Observe transition samples $x_t, a_t \xrightarrow{r_t} x_{t+1}$

Update:

$$Z(x_t, a_t) = (1 - \alpha_t) Z(x_t, a_t) + \alpha_t \Pi_C (r_t + \gamma Z(x_{t+1}, \pi_Z(x_{t+1}))$$

Theorem

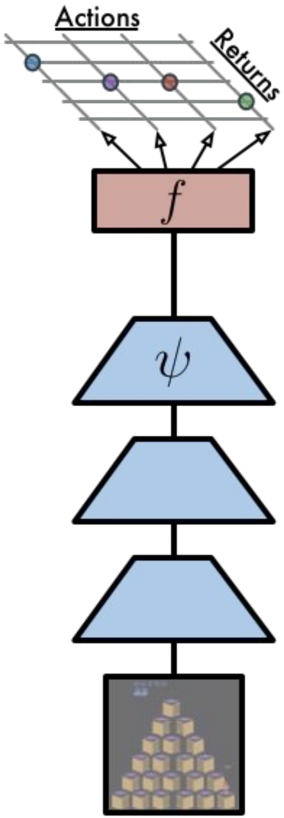
Under the same assumption as for Q-learning,
assume there is a unique optimal policy π^*,
then $Z \to Z_n^{\pi^*}$ and the resulting policy is optimal.

[Rowland et al., 2018]

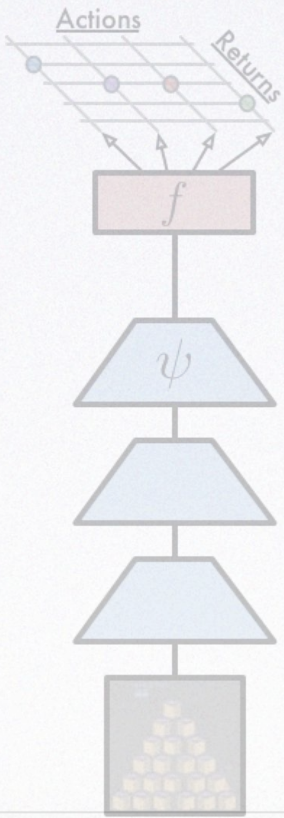
DeepRL implementation

DQN

[Mnih et al., 2013]

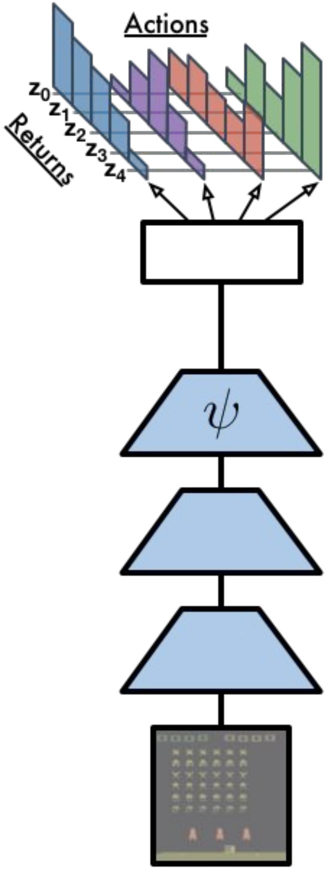


DQN

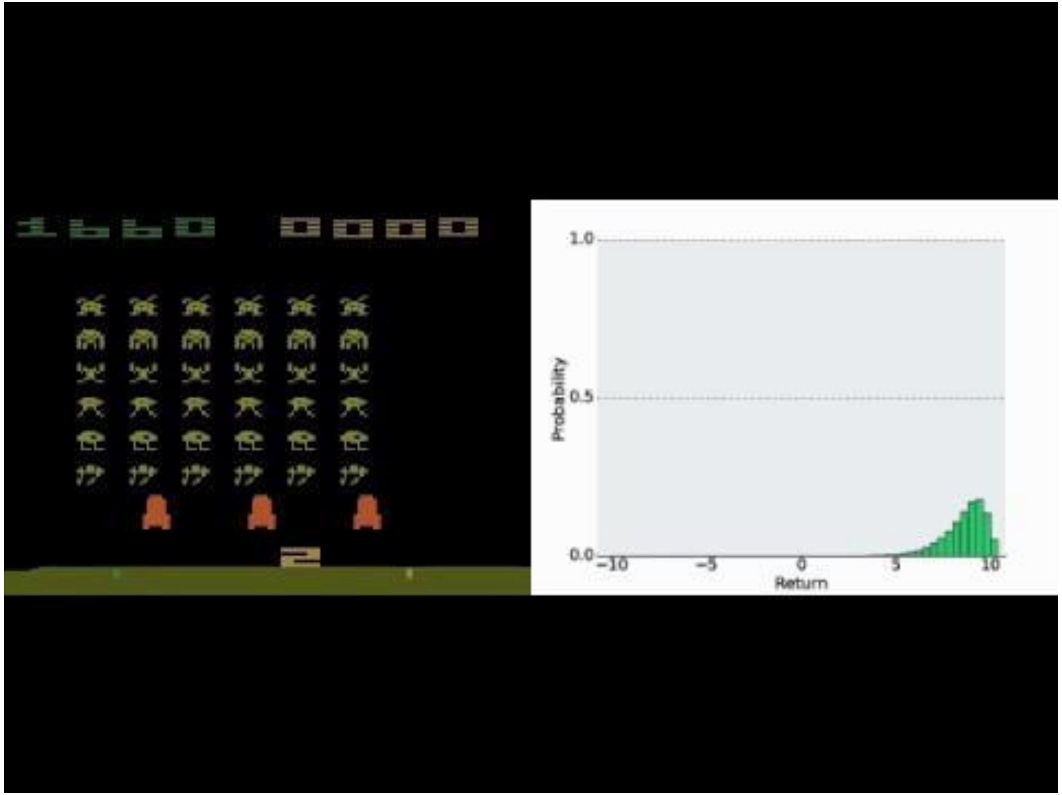


Categorical DQN

[Bellemare et al., 2017]

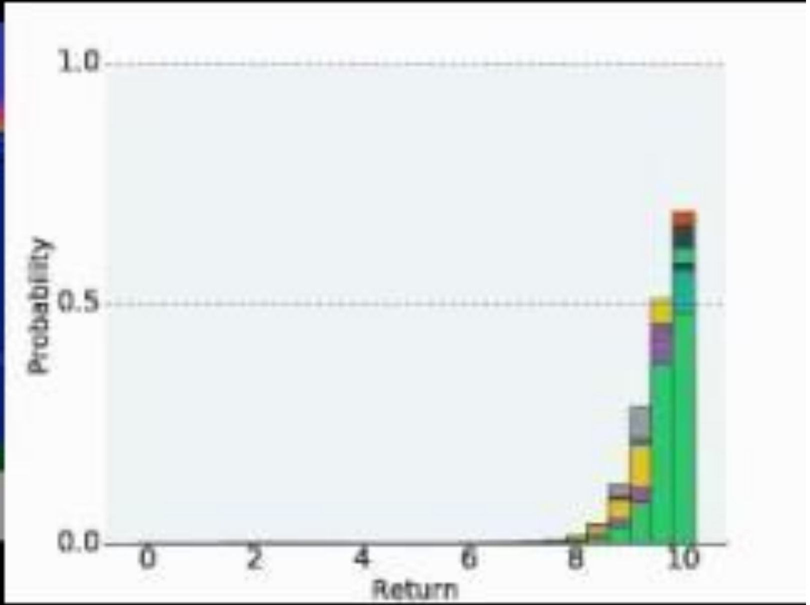


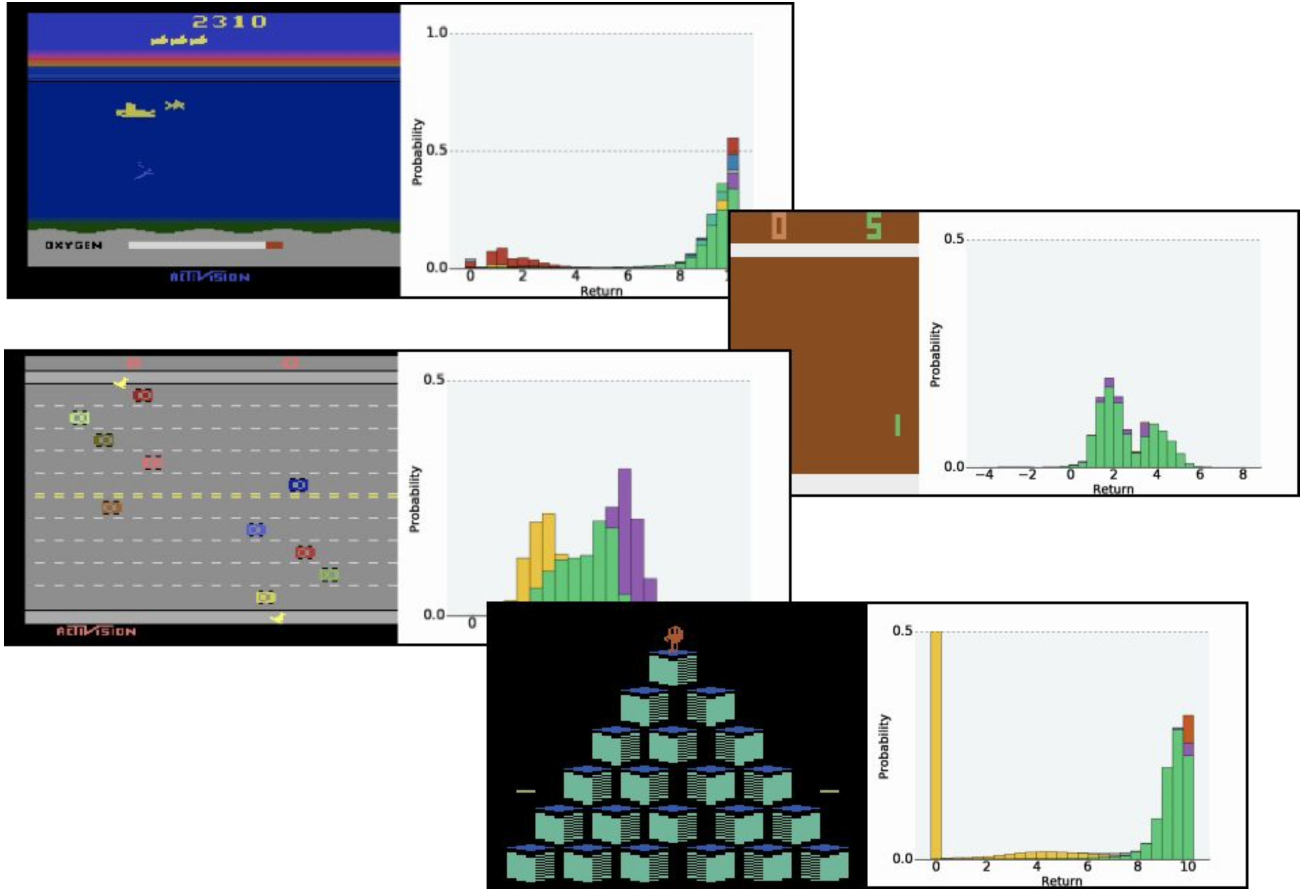
Categorical DQN



Randomness from future choices


20840
OXYGEN
ACTIVISION


1.0
0.5
0.0
Probability
0
2
4
6
8
10
Return



2310
OXYGEN
ACTIVISION
1.0
0.5
0.0
Probability
0
2
4
6
8
Return
0
5
0.5
0.0
Probability
−4
−2
0
2
4
6
8
Return
0.5
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Probability
0
ACTIVISION
0.5
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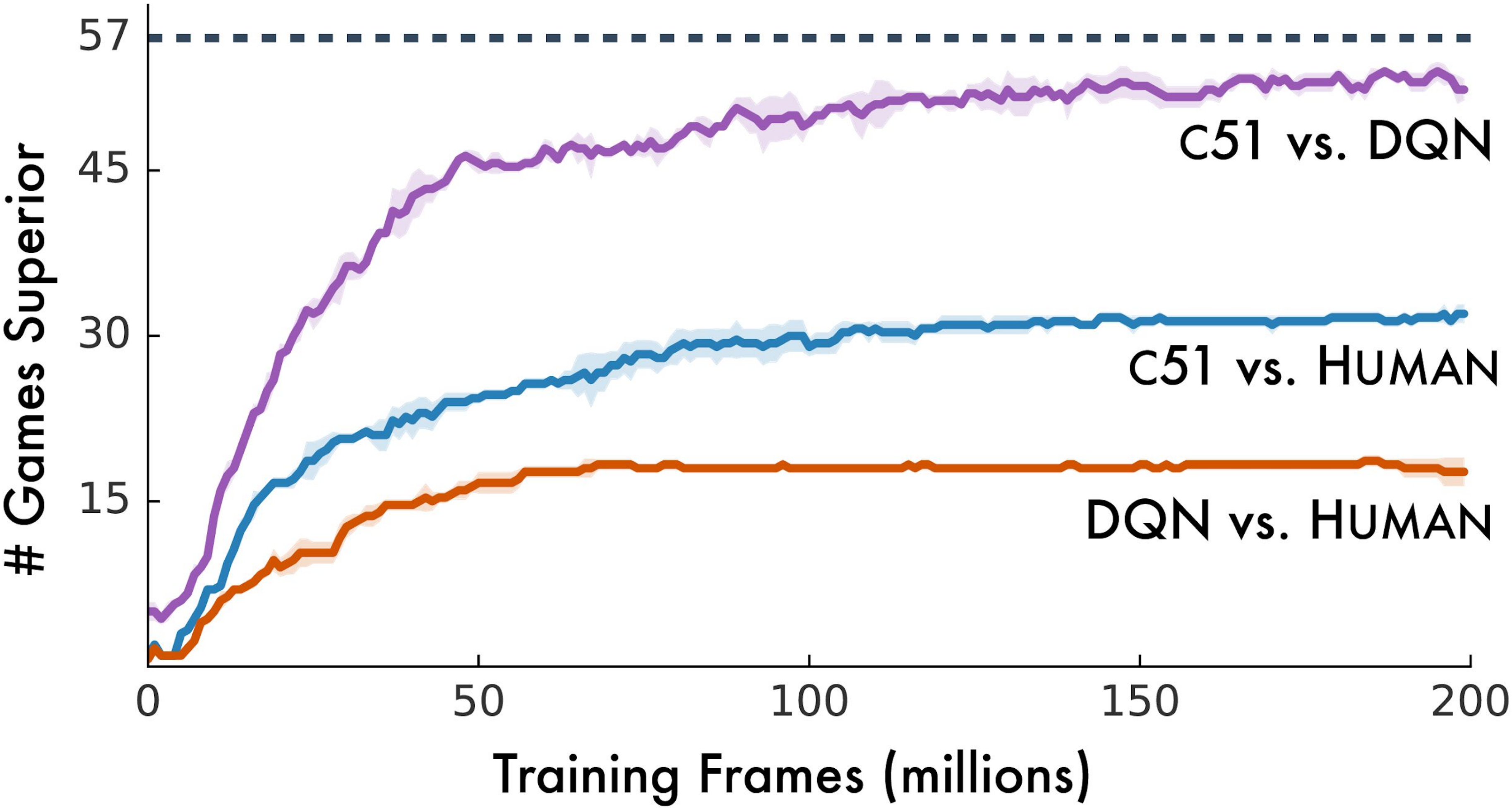
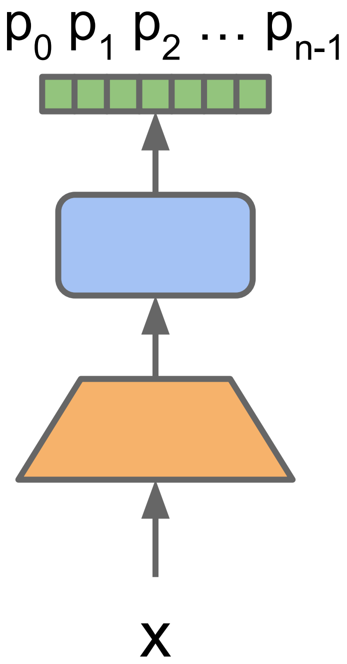
Probability
0
2
4
6
8
10
Return

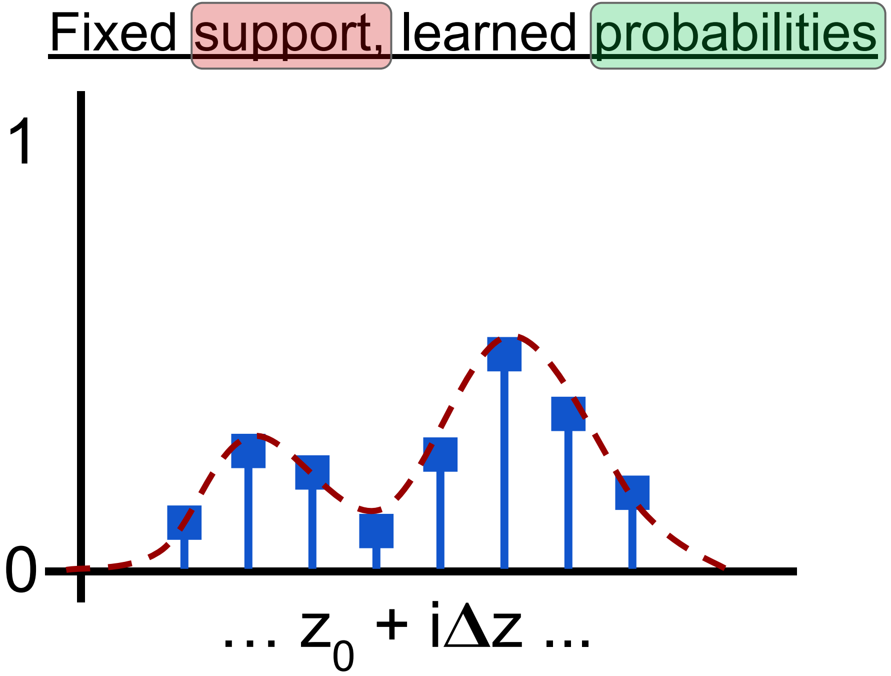

57
45
30
15
Games Superior
C51 vs. DQN
C51 vs. HUMAN
DQN vs. HUMAN
0
50
100
150
200
Training Frames (millions)

Results on 57 games Atari 2600

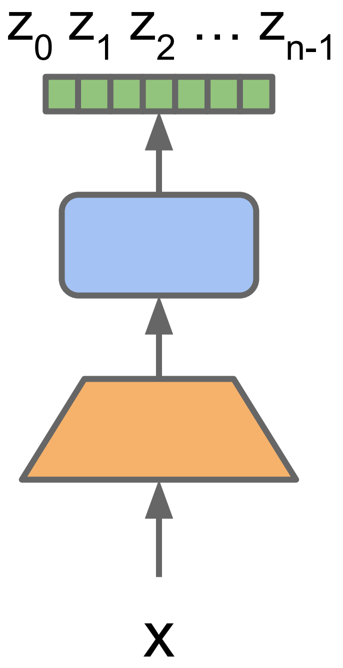
	Mean	Median	>human
DQN	228%	79%	24
Double DQN	307%	118%	33
Dueling	373%	151%	37
Prio. Duel.	592%	172%	39
C51	701%	178%	40

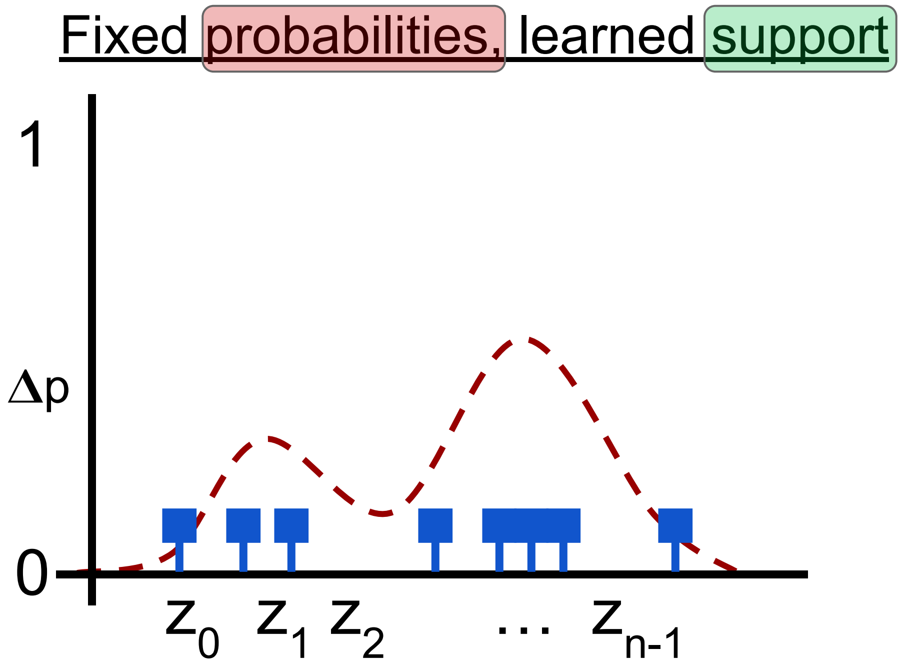
Categorical representation



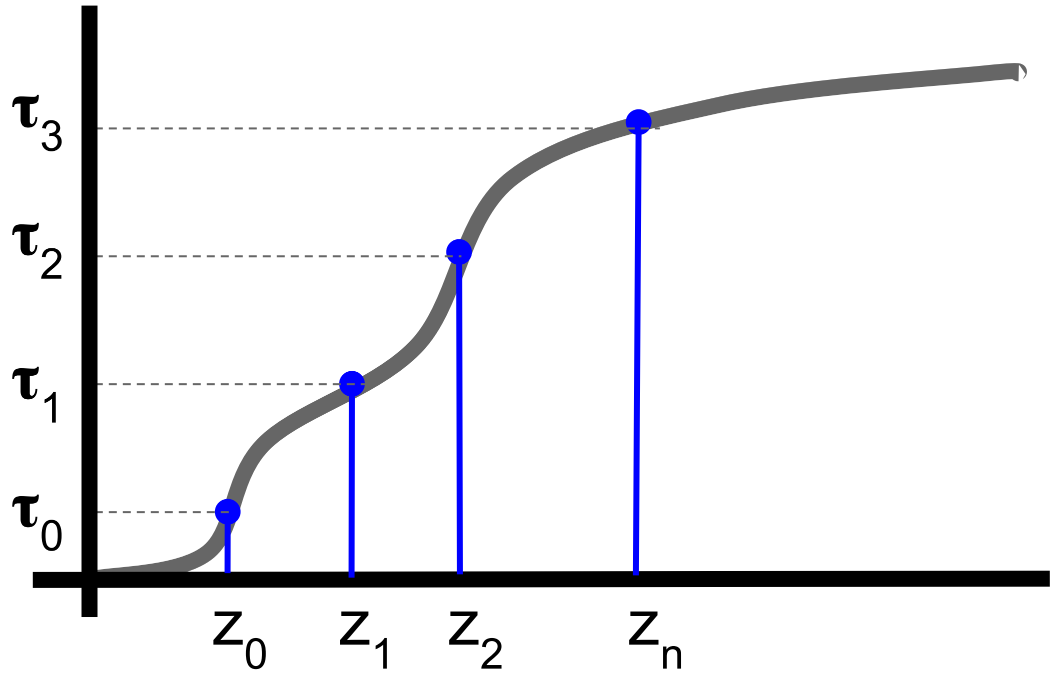


Quantile Regression Networks

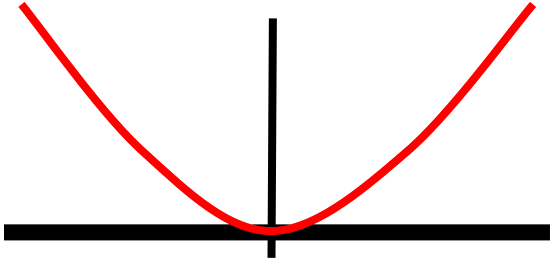




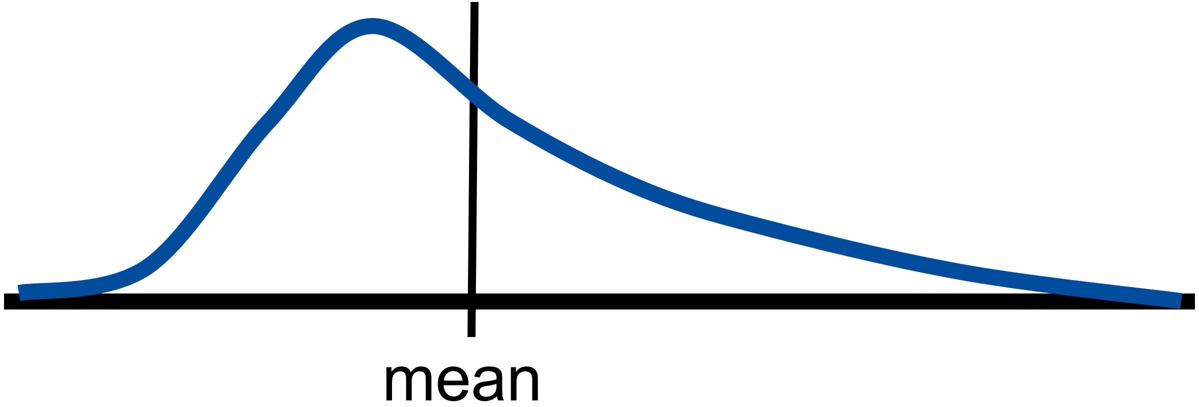
Inverse CDF learnt by Quantile Regression



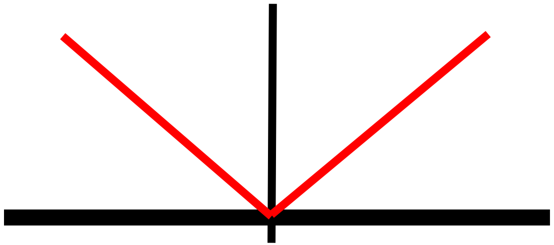
l2-regression



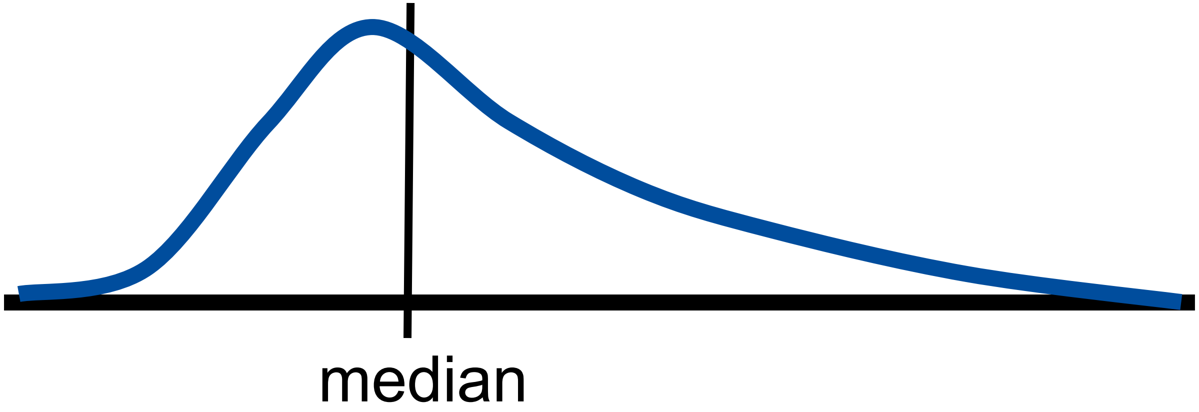
$$loss = x^2$$



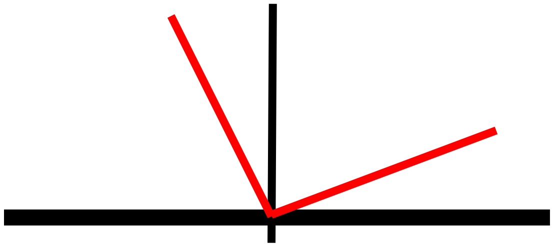
l1-regression



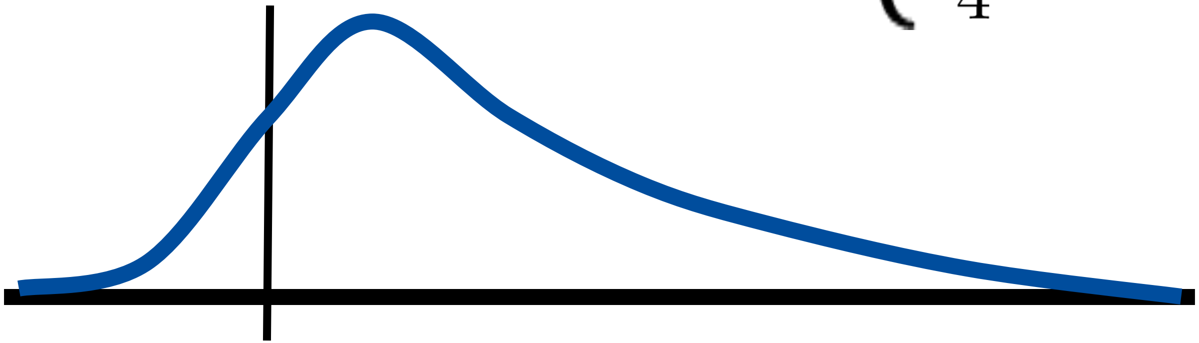
$$loss = |x|$$



¼-quantile-regression

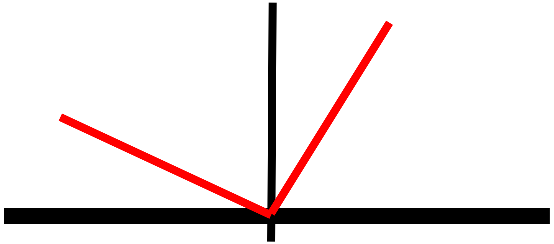


$$loss = \begin{cases} \frac{1}{4}x, & \text{for } x \geq 0 \\ -\frac{3}{4}x, & \text{for } x < 0 \end{cases}$$

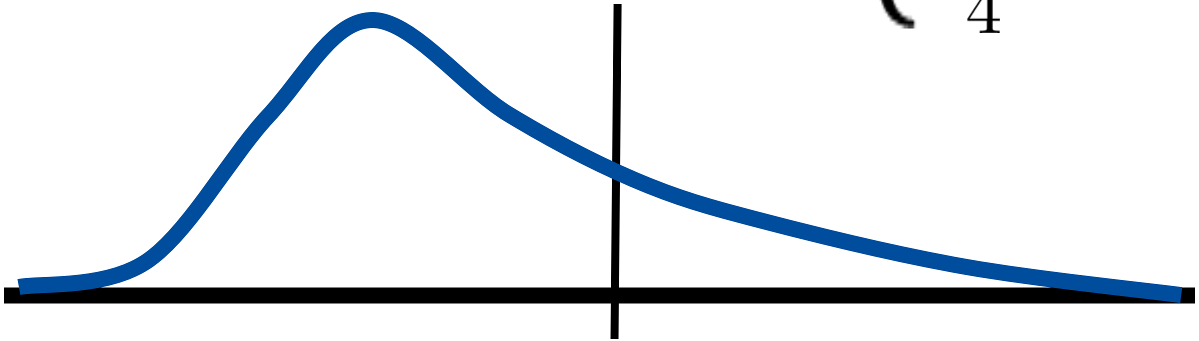


¼-quantile

¾-quantile-regression

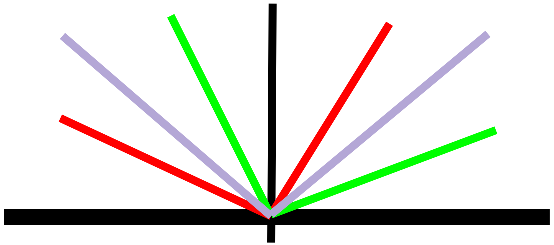


$$loss = \begin{cases} \frac{3}{4}x, & \text{for } x \geq 0 \\ -\frac{1}{4}x, & \text{for } x \geq 0 \end{cases}$$

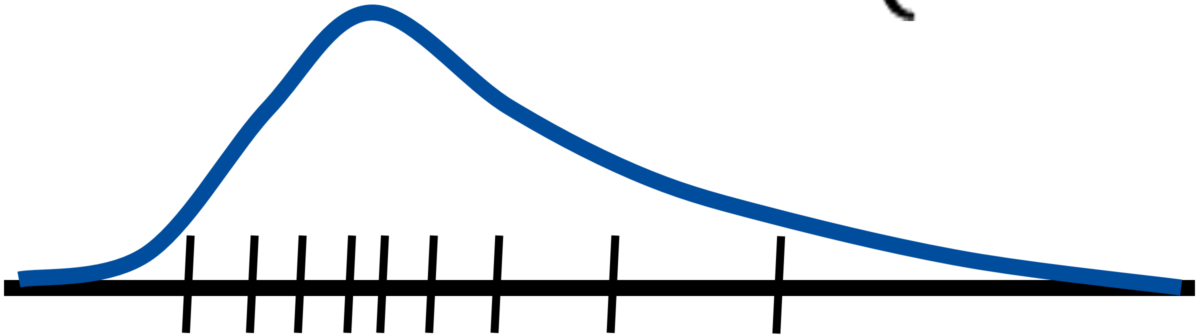


¾-quantile

many-quantiles-regression



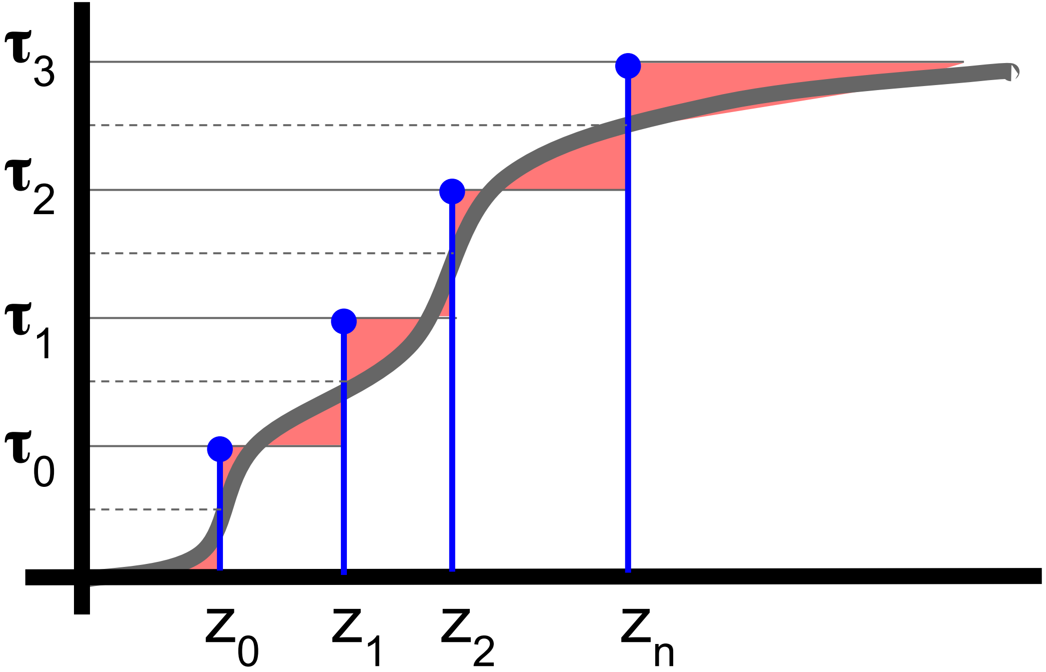
$$loss = \begin{cases} \tau x, & \text{for } x \geq 0 \\ (\tau - 1)x, & \text{for } x \geq 0 \end{cases}$$



many-quantiles

Quantile Regression = projection in Wasserstein!

(on a uniform grid)



QR distributional Bellman operator

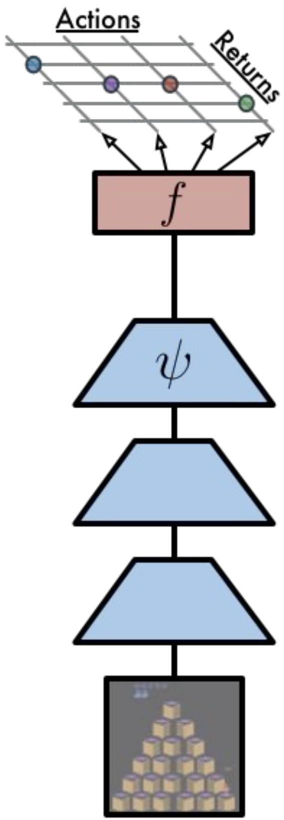
Theorem: $\boxed{\Pi_{QR} T^{\pi} \text{ is a contraction (in Wasserstein)}}$ [Dabney et al., 2018]

Intuition: quantile regression = projection in Wasserstein

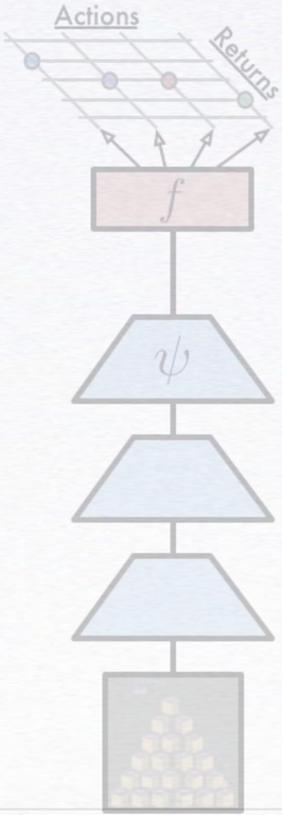
Reminder:

- T^{π} is a contraction (both in Cramer and Wasserstein)
- $\Pi_n T^{\pi}$ is a contraction (in Cramer)

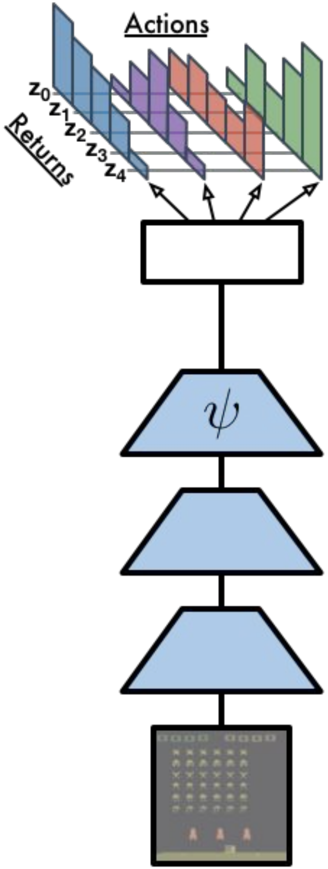
DQN



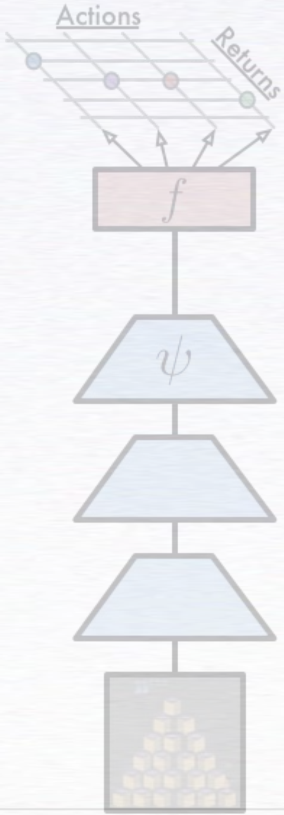
DQN



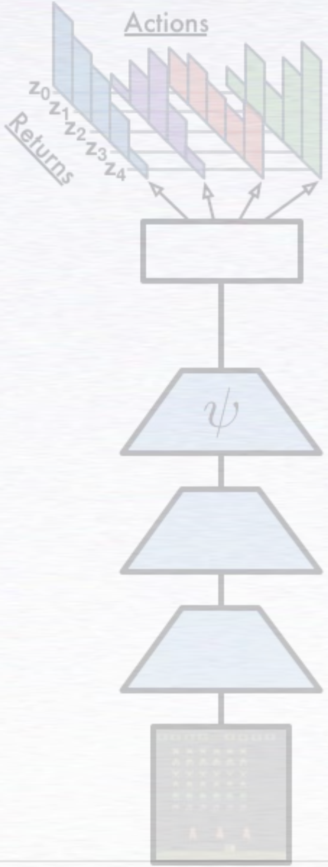
C51



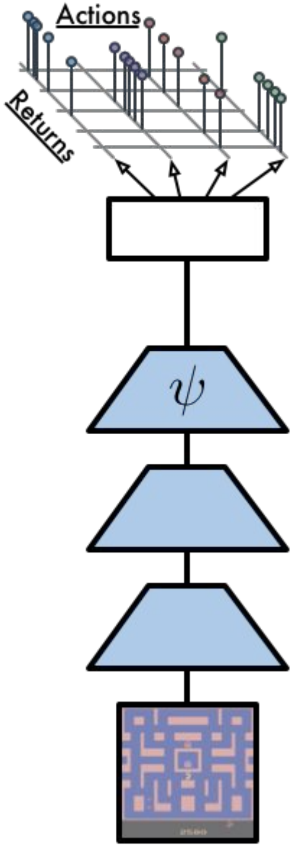
DQN



C51



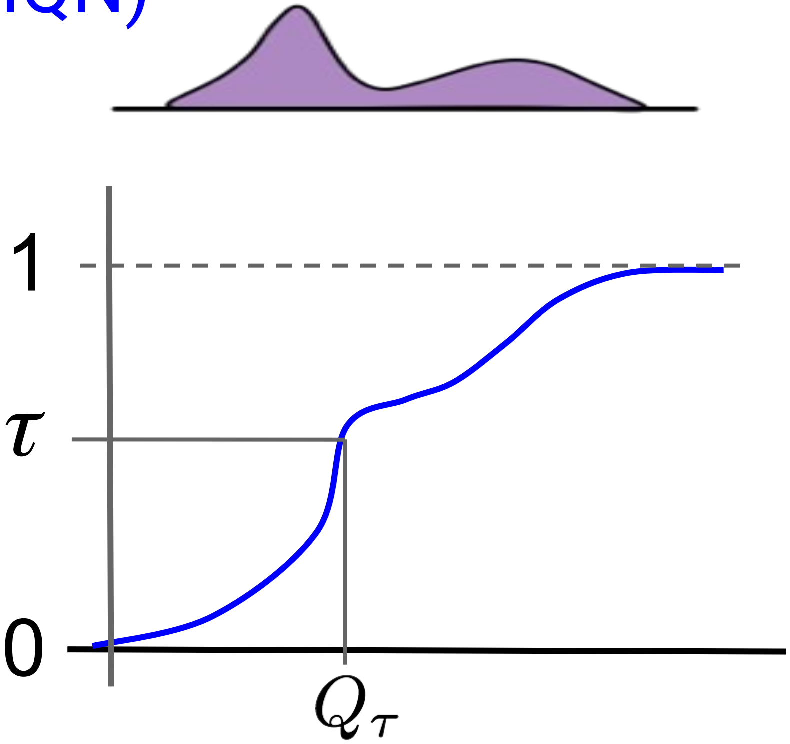
QR-DQN



Quantile-Regression DQN

	Mean	**Median**
DQN	228%	79%
Double DQN	307%	118%
Dueling	373%	151%
Prio. Duel.	592%	172%
C51	701%	178%
QR-DQN	864%	193%

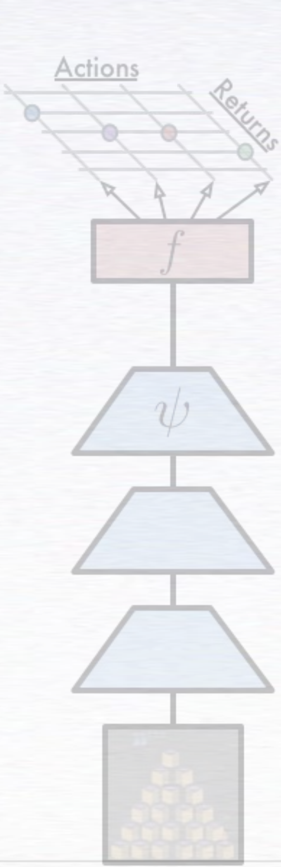
Implicit Quantile Networks (IQN)



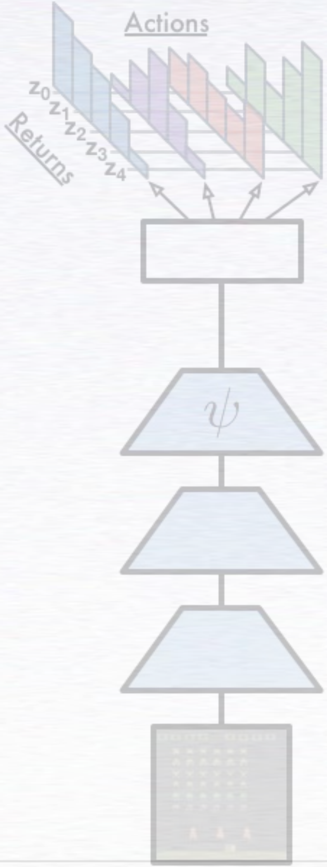
Learn a parametric inverse CDF

$$\tau \mapsto F_Z^{-1}(\tau)$$

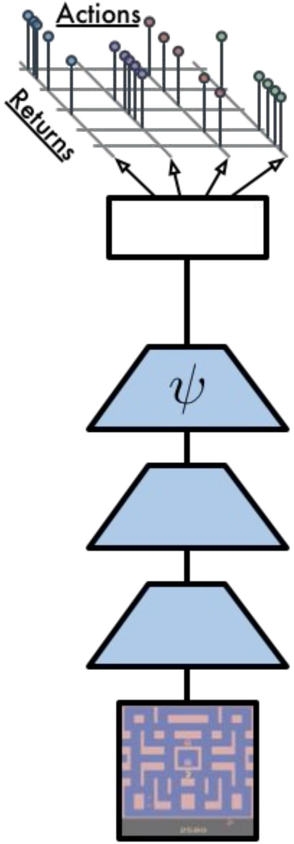
DQN



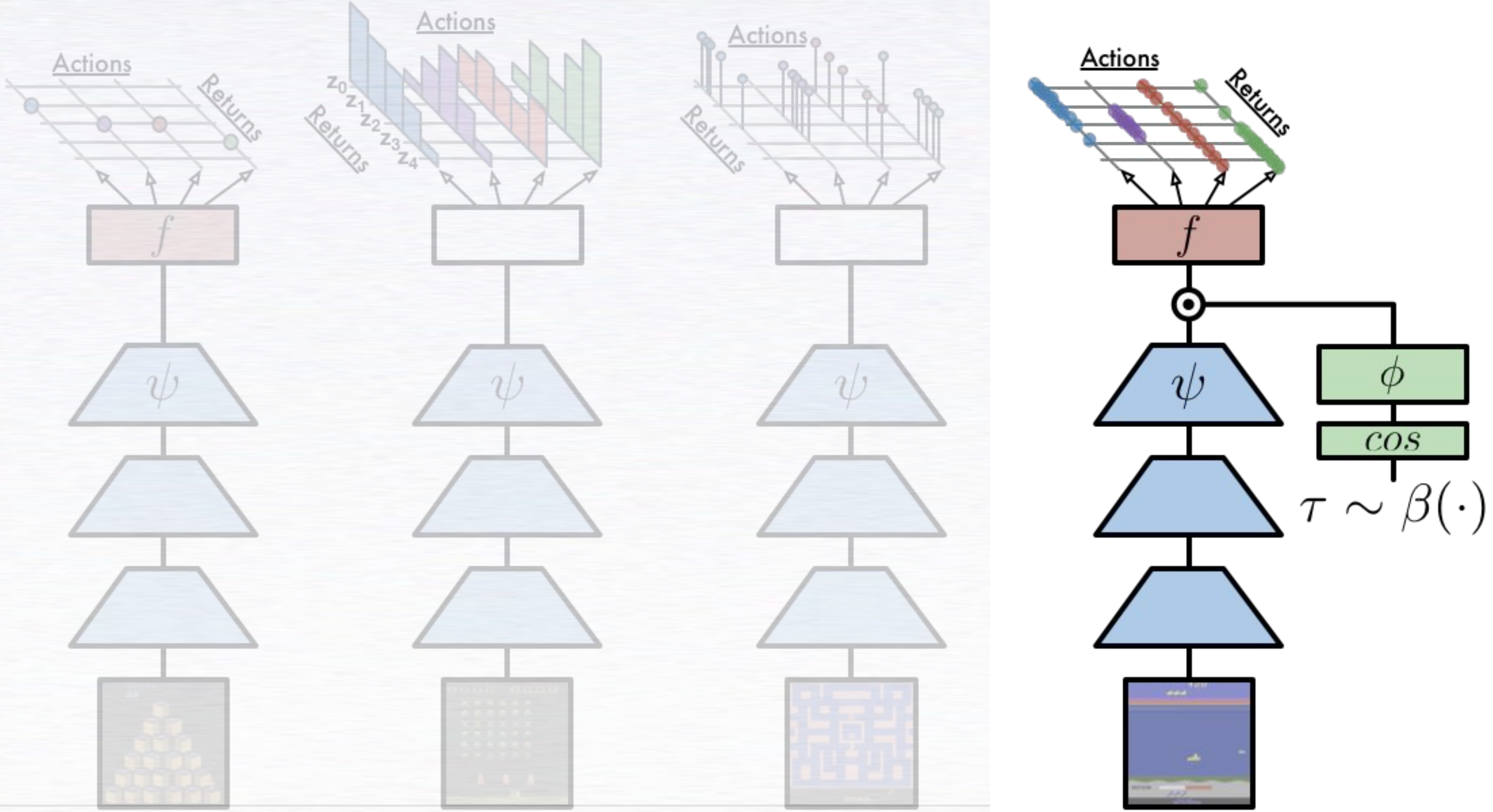
C51



QR-DQN



DQN C51 QR-DQN **IQN**

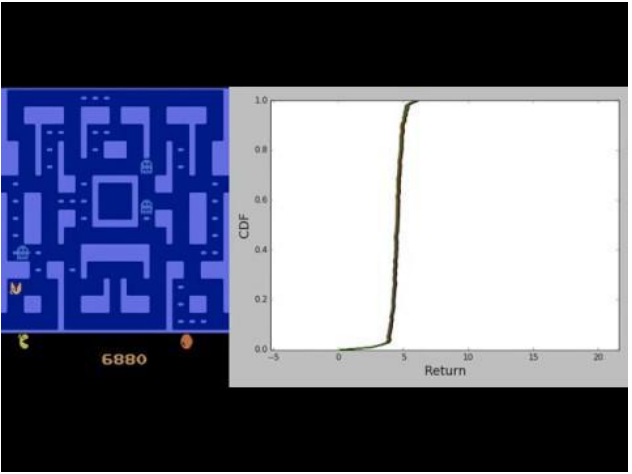


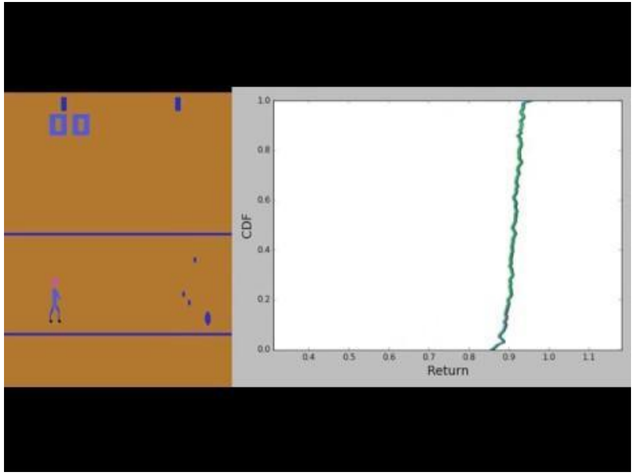
Implicit Quantile Networks for TD

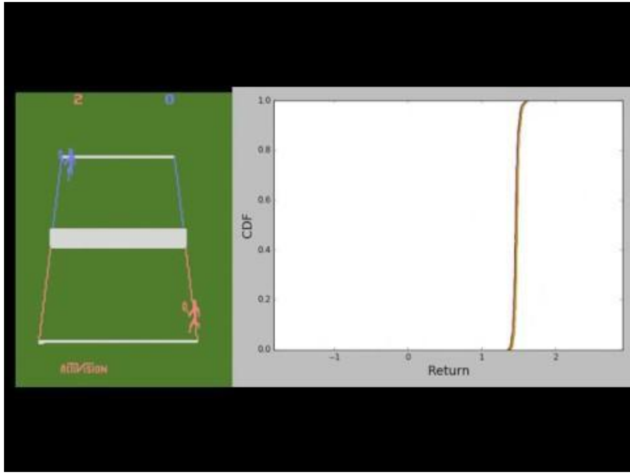
$$\tau \sim \mathcal{U}[0,1], \qquad z = Z_\tau(x_t, a_t)$$
$$\tau' \sim \mathcal{U}[0,1], \qquad z' = Z_\tau(x_{t+1}, a^*)$$

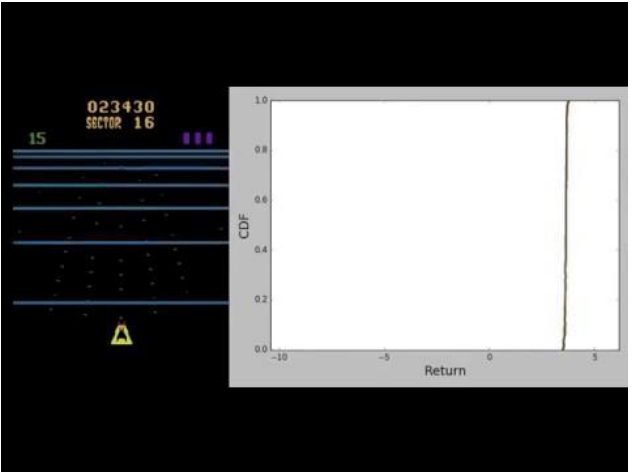
$$\delta_t = r_t + \gamma z' - z$$

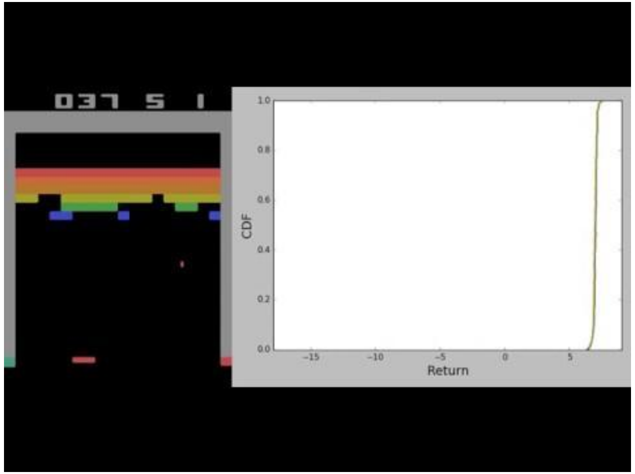
$$\text{QR loss: } \rho_\tau(\delta) = \delta(\tau - \mathbb{I}_{\delta<0})$$

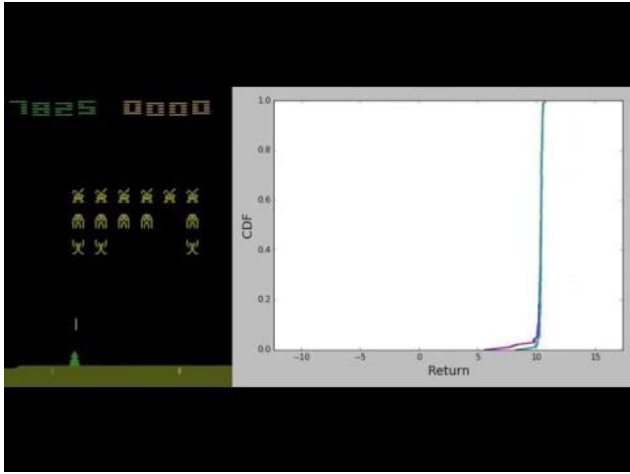

6880
CDF
Return


CDF
Return


ACTIVISION
CDF
Return


023430
SECTOR 16
15
CDF
Return


CDF
Return


CDF
Return

Implicit Quantile Networks

	Mean	Median	Human starts
DQN	228%	79%	68%
Prio. Duel.	592%	172%	128%
C51	701%	178%	116%
QR-DQN	864%	193%	153%
IQN	**1019%**	**218%**	**162%**

Almost as good as SOTA (Rainbow/Reactor) which combine prio/dueling/categorical/...

What is going on?

- We learn these distributions, but in the end **we only use their mean**

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Non-trivial **interactions between deep learning and RL**:

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  - Can better disambiguate between different states (state aliasing)
- Density estimation instead of l2-regressions
  - Express RL in terms of usual tools in deep learning

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- Density estimation instead of l2-regressions
  - Express RL in terms of usual tools in deep learning

Now maybe we could start using those distributions? (e.g, risk-sensitive control, exploration, …)

Thanks!

References:

- *A distributional perspective on reinforcement learning*, Bellemare, Dabney, Munos, ICML 2017
- *An Analysis of Categorical Distributional Reinforcement Learning*, Rowland, Bellemare, Dabney, Munos, Teh, AISTATS 2018
- *Distributional reinforcement learning with quantile regression*, Dabney, Rowland, Bellemare, Munos, AAAI 2018
- *Implicit Quantile Networks for Distributional Reinforcement Learning*, Dabney, Ostrovski, Silver, Munos, ICML 2018