

Inverse problems

High level formulation

- Forward process to “invert”

$$\mathbf{y} \rightarrow \mathbf{x} = g(\mathbf{y})$$

$$\hat{\mathbf{y}}(\mathbf{x}) \in g^{-1}(\mathbf{x})?$$

- Optimization-based inversion

$$\forall \mathbf{x}, \hat{\mathbf{y}}(\mathbf{x}) \in \arg \min_{\mathbf{y}} \underbrace{\left(\text{Loss}(\mathbf{x}, g(\mathbf{y})) + \text{Prior}(\mathbf{y}) \right)}_{E(\mathbf{x}, \mathbf{y}; \theta)}$$

- Model bricks: given (e.g., *physics*), engineered, and/or *learned*

Classic examples

Signal enhancement or completion

- Forward process: degradation and/or masking
- Prior: spatial/temporal regularity

$$\min_{\mathbf{y}} \left(\|\mathbf{x} - g(\mathbf{y})\|_2^2 + \mu \|\nabla \mathbf{y}\|_p^q \right) \text{ s.t. } \mathbf{y}|_{\partial D} = \mathbf{y}^*$$

Signal recovery

- Linear forward process: atom composition, random measurements
- Prior: sparsity

$$\min_{\mathbf{y}} \left(\|\mathbf{x} - A\mathbf{y}\|_2^2 + \mu \|\mathbf{y}\|_1 \right)$$

Less classic examples

Inverse graphics

- CGI: from scene model to photoreal images
- *Inverse rendering*: from real images to scene model (“graphic code”)
- Application: AR, VR, editing, retargeting, personalized models



[Tewari 2017]

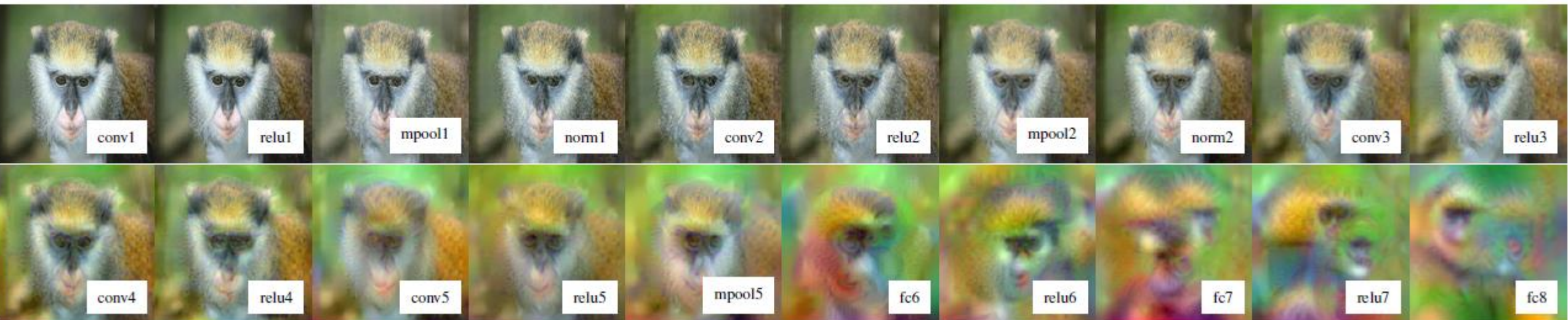
Less classic examples

“Neural” inverse problems

- Deep features: from signal to representations through feedforward neural net
- *Inverting*: from neural activations to NN input

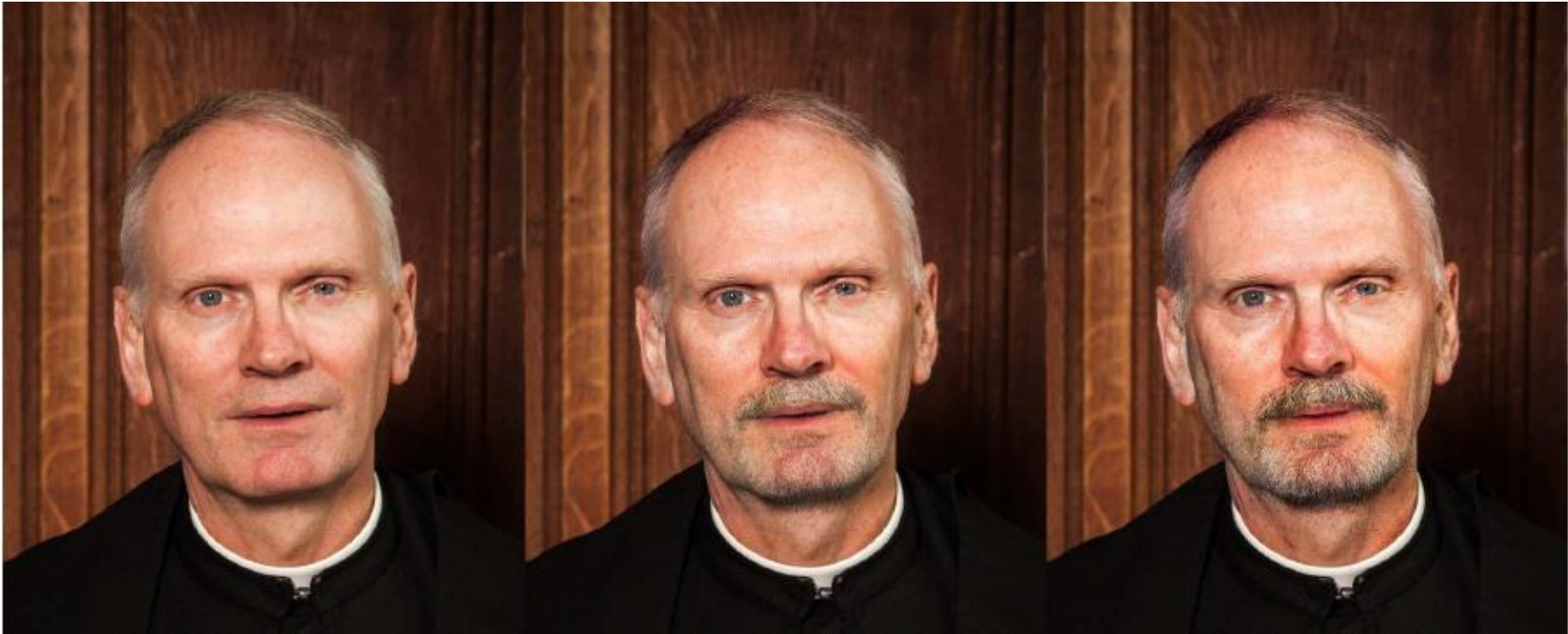
$$\min_{\mathbf{y}} \left(\|\mathbf{x} - \phi_{\ell_0}(\mathbf{y})\|_{\mathbb{F}}^2 + \mu \|\nabla \mathbf{y}\|_1 \right)$$

- Application: visualization, inspection, editing in feature domain



Less classic examples

“Neural” inverse problems



[Upchich 2017]

Learning and inverse problems?

Learn the model, solve by optimization

- Forward process, prior and loss can be learned
- Examples: blind deconvolution, trained MRFs
- Inference with a classic solver, *iterative* and *generic*

$$\forall \mathbf{x}, \text{Solver}(\mathbf{x}) = \text{Iter}^{\infty}(\mathbf{x}; \mathbf{y}^0) \approx \hat{\mathbf{y}}(\mathbf{x})$$

Train a direct solver

- From $\forall \mathbf{x}, \hat{\mathbf{y}}(\mathbf{x}) \in \arg \min_{\mathbf{y}} E(\mathbf{x}, \mathbf{y})$
- To $\forall \mathbf{x} \sim \mathbb{P}_X, f(\mathbf{x}; \mathcal{W}) \approx \hat{\mathbf{y}}(\mathbf{x})$

Neural inversion

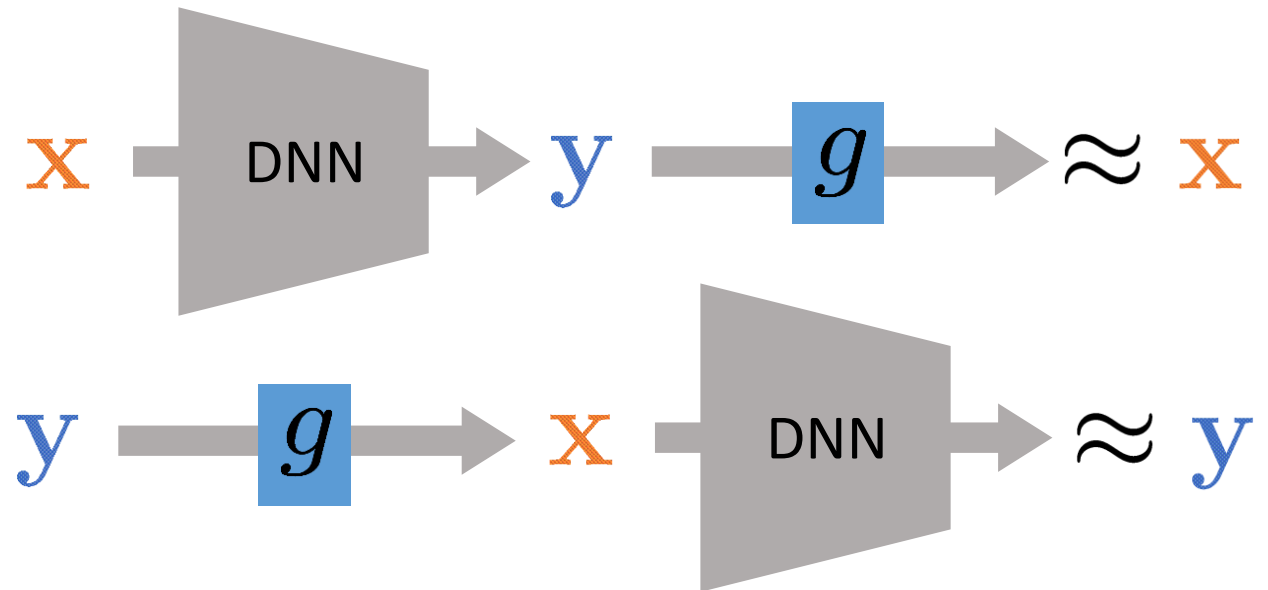
Train a DNN to regress solution *for plausible inputs*

$$\forall \mathbf{x} \sim \mathbb{P}_X, \text{DNN}(\mathbf{x}; \mathcal{W}) \approx \hat{\mathbf{y}}(\mathbf{x})$$

- Fixed complexity
- Possibly way faster
- Flexible in various ways
- Differentiable

Architecture?

Training?



Architectures

Your favorite DNN

- Exploit popular architectures, possibly pre-trained
- Popular convolutional and recurrent neural nets

Unrolling

- Mimic (possibly loosely) structure of iterative solver
- Each [non-linear ◦ linear] iteration becomes a trainable neural layer
- Fixed, smaller number of “iterations”
- But, way more freedom exploited through training

Training

Fully-supervised – Using Solver, reconstruction loss

$$\mathbf{x}^{(n)} \sim \mathbb{P}_X, \min_{\mathcal{W}} \sum_{n=1}^N \|\text{Solver}(\mathbf{x}^{(n)}) - \text{DNN}(\mathbf{x}^{(n)}; \mathcal{W})\|_2^2$$

Self-supervised – Only “synthetic data”, reconstruction loss

$$\mathbf{y}^{(n)} \sim \mathbb{P}_Y, \min_{\mathcal{W}} \sum_{n=1}^N \|\mathbf{y}^{(n)} - \text{DNN}(g(\mathbf{y}^{(n)}); \mathcal{W})\|_2^2$$

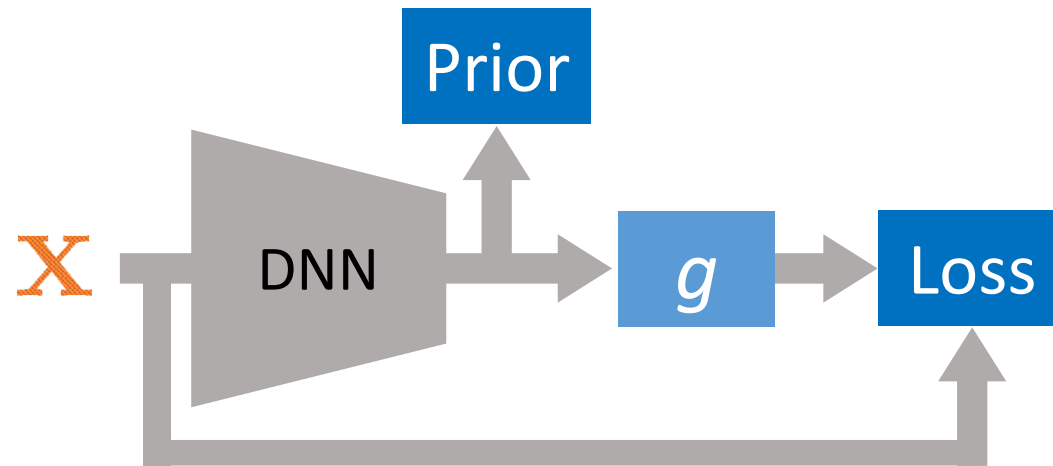
Unsupervised – Original objective function as training loss

$$\mathbf{x}^{(n)} \sim \mathbb{P}_X, \min_{\mathcal{W}} \sum_{n=1}^N E(\mathbf{x}^{(n)}, \text{DNN}(\mathbf{x}^{(n)}; \mathcal{W}); \boldsymbol{\theta})$$

Unsupervised training

Unsupervised – Original objective function as training loss

$$\mathbf{x}^{(n)} \sim \mathbb{P}_X, \min_{\mathcal{W}} \sum_{n=1}^N \text{Loss}(\mathbf{x}^{(n)}, g \circ \text{DNN}(\mathbf{x}^{(n)}, \mathcal{W})) + \text{Prior}(\text{DNN}(\mathbf{x}^{(n)}, \mathcal{W}))$$

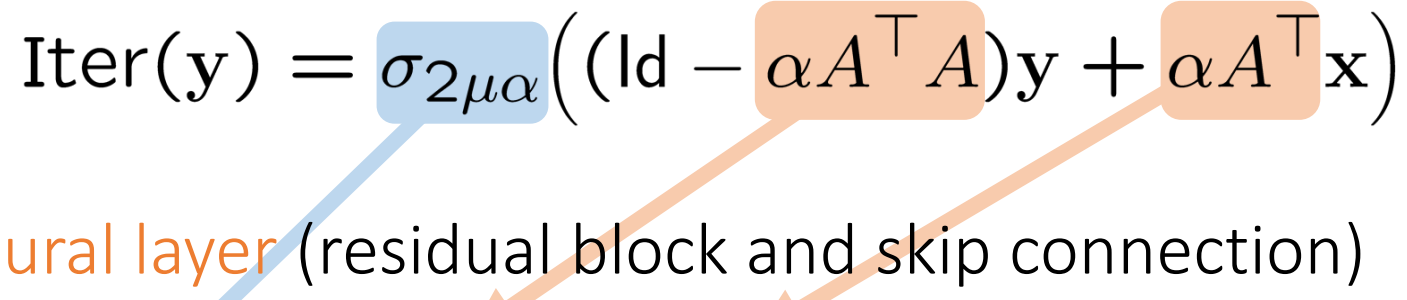


Unrolling example

Sparse coding: LISTA [Gregor 2010]

$$\min_{\mathbf{y}} (0.5 \|\mathbf{x} - A\mathbf{y}\|_2^2 + \mu \|\mathbf{y}\|_1)$$

Iterative solver: Iterative Soft Thersholding Alg. (ISTA)

$$\text{Iter}(\mathbf{y}) = \sigma_{2\mu\alpha} \left((\text{Id} - \alpha A^\top A) \mathbf{y} + \alpha A^\top \mathbf{x} \right)$$


Learned neural layer (residual block and skip connection)

$$f_k(\mathbf{y}) = \sigma \left((\text{Id} - V_k) \mathbf{y} + W_k^\top \mathbf{x} \right), \mathcal{W} = \{(W_k, V_k)\}_k$$

- Fully supervised training
- The deeper, the better the approximation
- Modest speed-up

Unsupervised neural inversion

Personalized 3D face model: Tewari *et al.* 2017

Encoder-decoder with differentiable rendering layer

Artistic style transfer: Ulyanov *et al.* 2016, Johnson *et al.* 2016

Encoder-decoder with “perceptual loss”

Flexible style transfer: Puy & Pérez 2019

Unrolling descent, run time flexibility

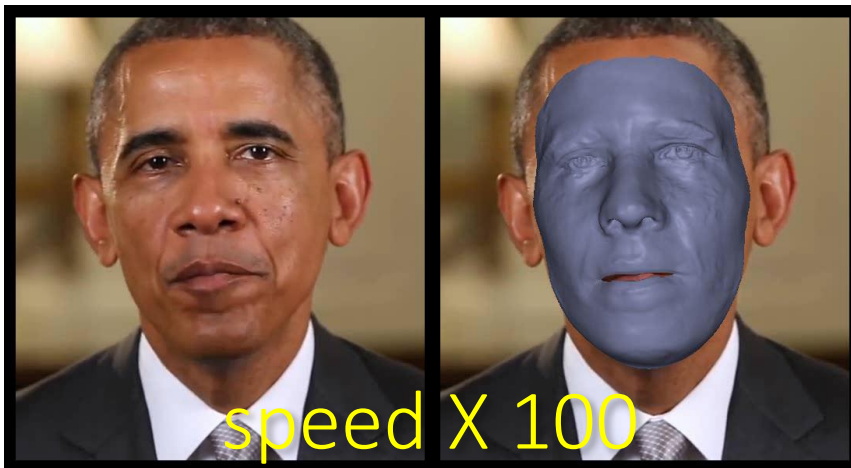
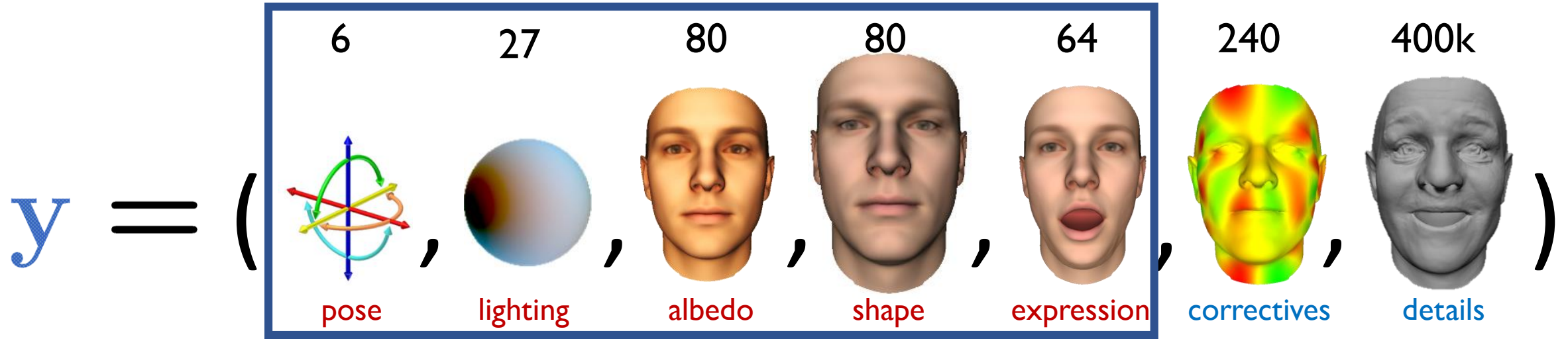


Photo-real face rendering



$$\mathbf{y} \in \mathbb{R}^{257} \rightarrow g(\mathbf{y}) = \text{[face image]} = \text{[base face image]} \otimes \left[\text{[correctives map]} \circ \text{[details map]} \right]$$

[Garrido 2016]

Invert rendering

to obtain animatable personalized 3D rig

$$\mathbf{y} = (\text{pose}, \text{lighting}, \text{albedo}, \text{shape}, \text{expression}, \text{correctives}, \text{details})$$

The vector \mathbf{y} is composed of the following components:

Component	Dimensions
pose	6
lighting	27
albedo	80
shape	80
expression	64
correctives	240
details	400k

$$\mathbf{x} = \text{Image} \rightarrow \mathbf{y} \in \mathbb{R}^{257}?$$

[Garrido 2016]

Invert rendering

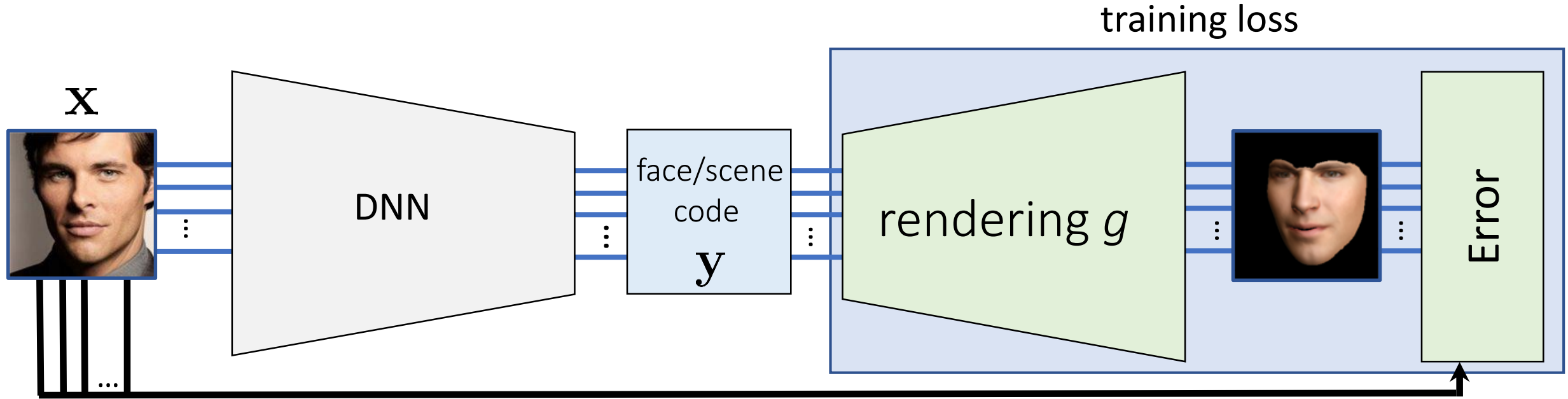
To obtain animatable personalized 3D rig

$$y = \left(\begin{array}{c} 6 \\ \text{pose} \end{array}, \begin{array}{c} 27 \\ \text{lighting} \end{array}, \begin{array}{c} 80 \\ \text{albedo} \end{array}, \begin{array}{c} 80 \\ \text{shape} \end{array}, \begin{array}{c} 64 \\ \text{expression} \end{array}, \begin{array}{c} 240 \\ \text{correctives} \end{array}, \begin{array}{c} 400k \\ \text{details} \end{array} \right)$$



[Garrido 2016]

Fast DNN solver



$$\text{Loss} = \left\| \begin{array}{c} \text{observed} \end{array} - \begin{array}{c} \text{rendered} \end{array} \right\|^2 + \left\| \begin{array}{c} \text{2D landmarks} \end{array} - \begin{array}{c} \text{correspondences} \end{array} \right\|^2$$

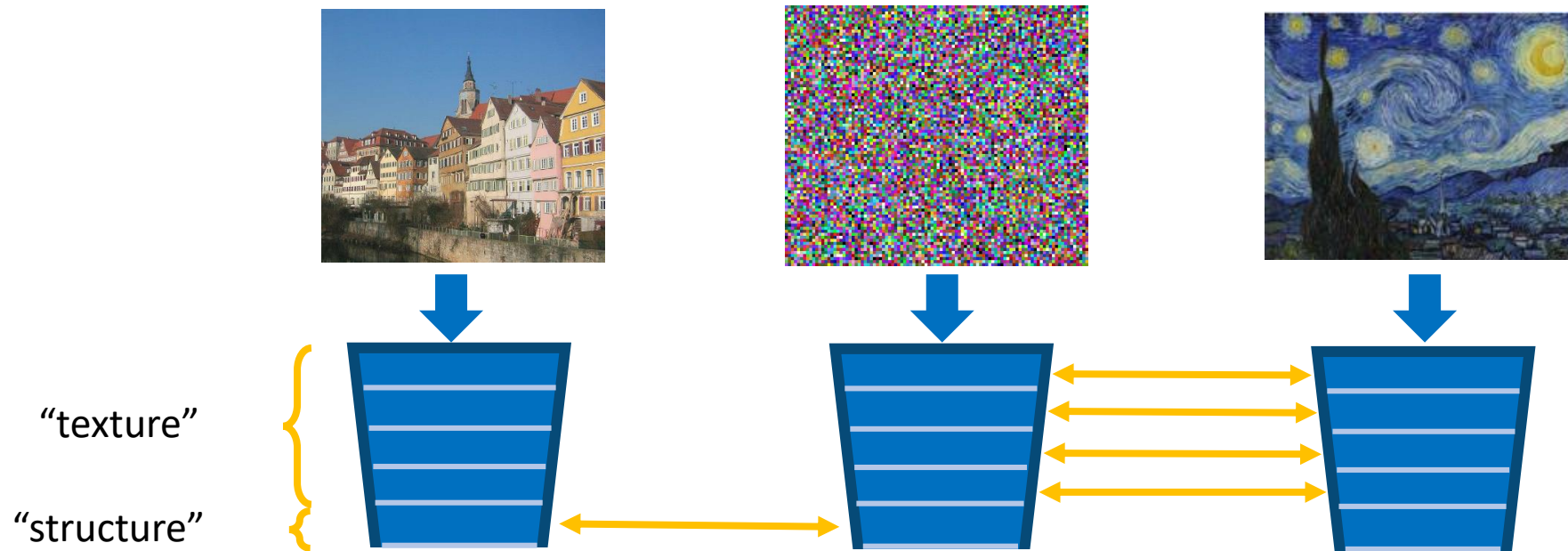
observed rendered 2D landmarks correspondences

[Tewari 2017-2018]

“Artistic” Style transfer

retain structure, imitate texture

$$\min_{\mathbf{y}} \left(\|\mathbf{x} - \phi_{\ell_0}(\mathbf{y})\|_{\mathbb{F}}^2 + \lambda \sum_{\ell \in \mathcal{L}_{\text{sty}}} \|G_{\ell} - \phi_{\ell}(\mathbf{y})^{\top} \phi_{\ell}(\mathbf{y})\|_{\mathbb{F}}^2 \right)$$



[Gatys 2015-2016]

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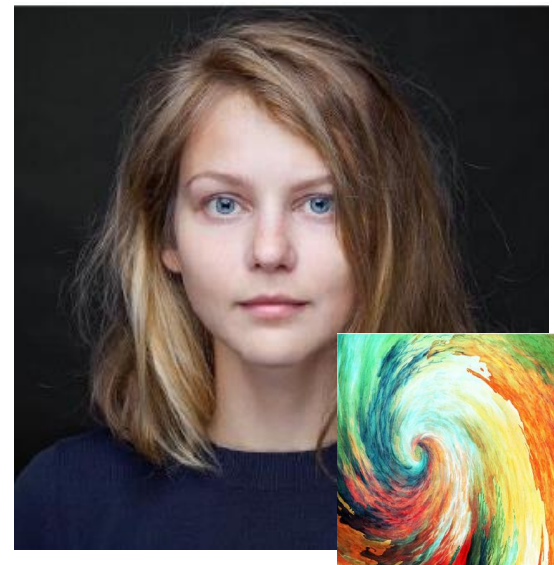
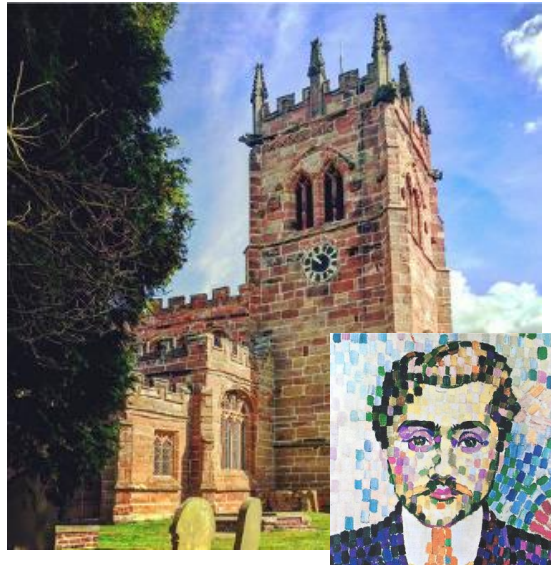


[Gatys 2015-2016]

Fast artistic style transfer

[Ulyanov 2016, Johnson 2016] and successors

- Convolutional encoder-decoder architectures
- Unsupervised training *for specified paintings*

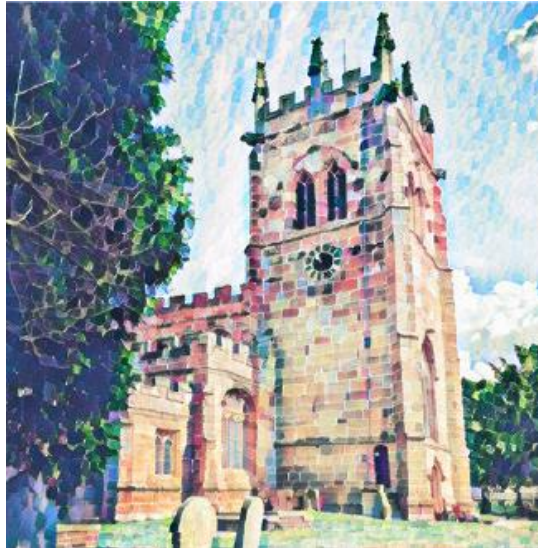


[Ulyanov 2017]

Fast artistic style transfer

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- Convolutional encoder-decoder architectures
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[Ulyanov 2017]

Fast flexible style transfer [Puy 2019]

Unrolling (part of) gradient descent

$$E(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \phi_{\ell_0}(\mathbf{y})\|_F^2 + \mu \|\nabla \mathbf{y}\|_1 \\ + \sum_{\ell \in \mathcal{L}_{\text{sty}}} \lambda_{\ell} \|G_{\ell} - \phi_{\ell}(\mathbf{y})^{\top} \phi_{\ell}(\mathbf{y})\|_F^2$$

- One layer mimics one step $\mathbf{y} \leftarrow \mathbf{y} - \alpha \nabla E_{\text{sty}}(\mathbf{x}, \mathbf{y})$

$$\mathbf{y}_k = \mathbf{y}_{k-1} - f_k\left(\mathbf{x}, \mathbf{y}_{k-1}; \mathcal{W}_k, \{\lambda_{\ell}, G_{\ell}\}_{\ell \in \mathcal{L}_{\text{sty}}}\right)$$

modifiable at *run time*

- Unsupervised training

Runtime restructuring

Choose style, mix styles, tune stylization intensity or scale



Runtime restructuring

Add new regularizers via proximal operator, e.g. for photorealism

$$\mathbf{y}_k = \mathbf{y}_{k-1} - \text{Prox}_{\Omega} \left[f_k(\mathbf{x}, \mathbf{y}_{k-1}) \right]$$



Runtime restructuring

Add new regularizers via proximal operator, e.g. for photorealism

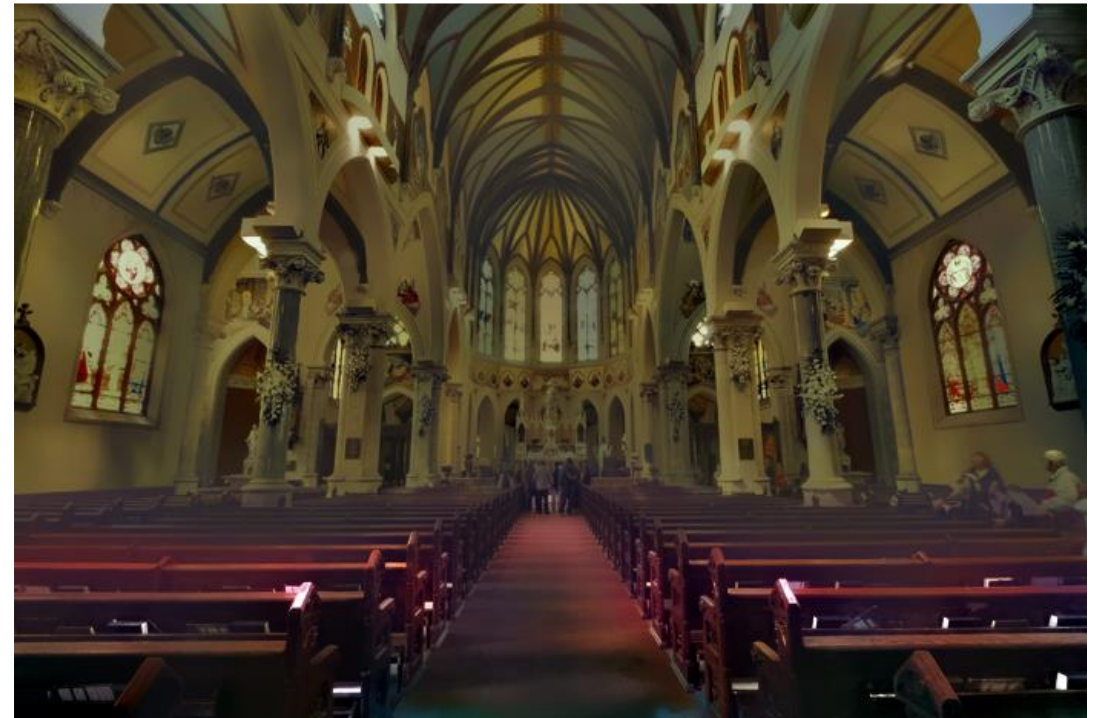
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Runtime restructuring

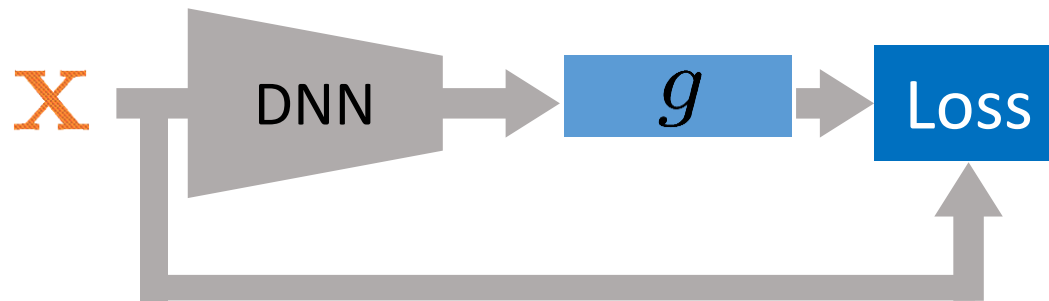
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$$\mathbf{y}_k = \mathbf{y}_{k-1} - \text{Prox}_{\Omega} \left[f_k(\mathbf{x}, \mathbf{y}_{k-1}) \right]$$



Perspectives

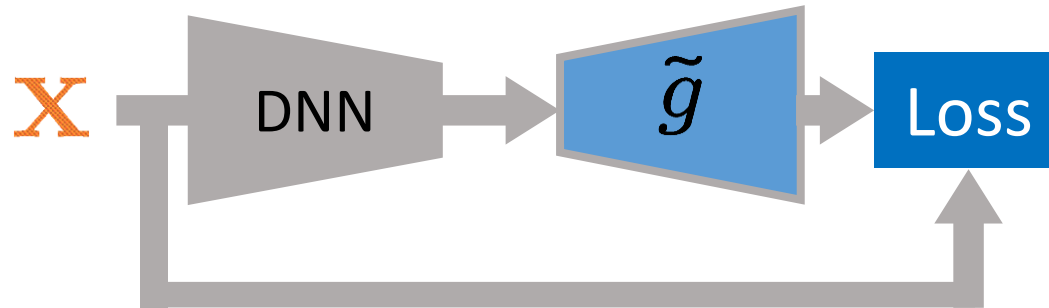
- Encoder-decoder view of unsupervised inversion: make forward process (partly) trainable, e.g. [Tewari 2018]



- *Invertible* NN [Ardizzone 2019] trained to mimic g , inversion for free
- Inversion of state-of-art GAN?
- Predict multiple relevant pre-images?

Perspectives

- Encoder-decoder view of unsupervised inversion: make forward process (partly) trainable, e.g. [Tewari 2018]



- *Invertible* NN [Ardizzone 2019] trained to mimic g , inversion for free
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- Predict multiple relevant pre-images?

Conclusion

Neural solvers for inverse problems

- fast, specialized, differentiable, possibly unsupervised, flexible
- can go beyond original model by learning
- applies to other optimization-based/variational problems

References

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