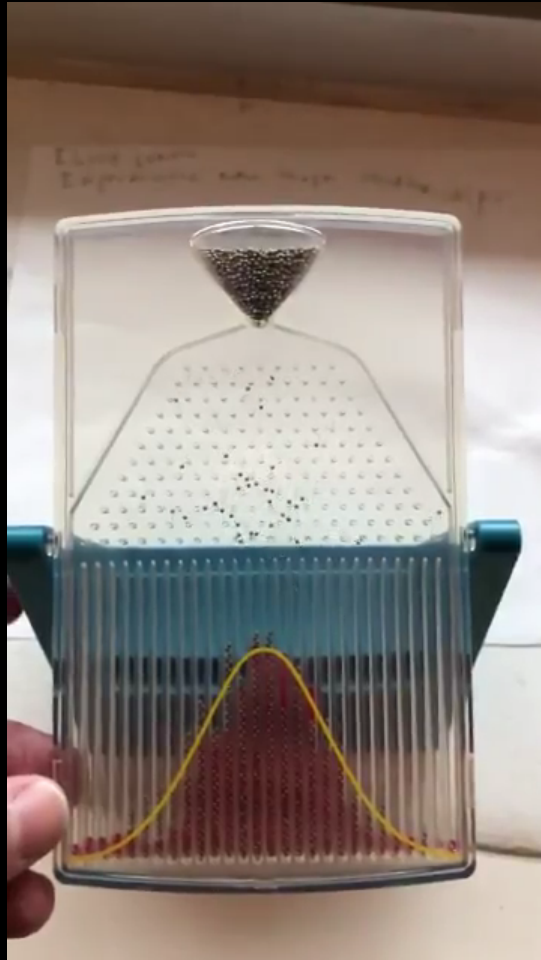


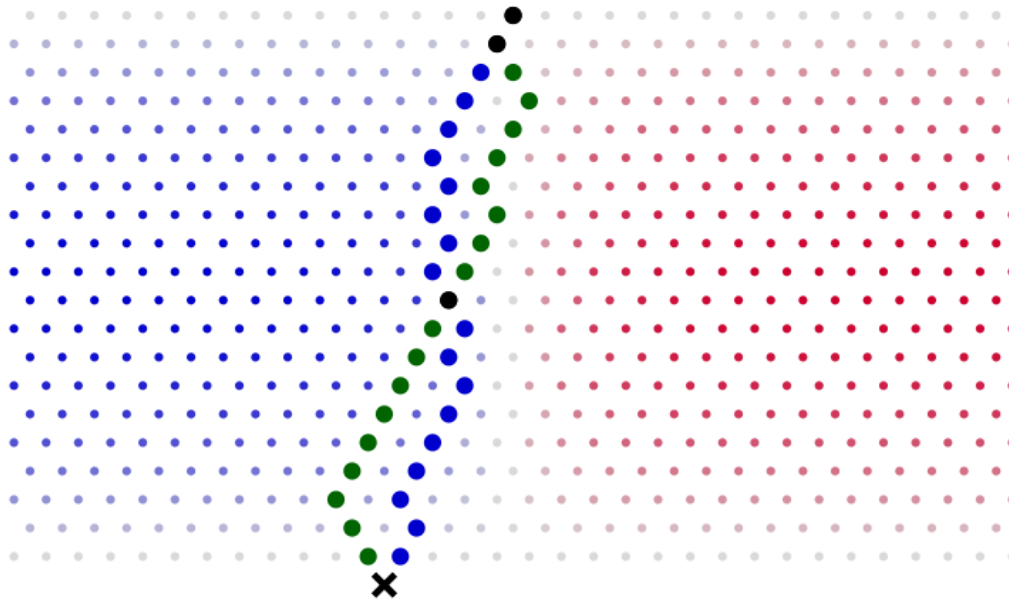
# Likelihood-free inference in Physical Sciences

Artificial Intelligence and Physics

Institut Pascal  
March 22, 2019

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[@glouppe](#)





The probability of ending in bin  $x$  corresponds to the total probability of all the paths  $z$  from start to  $x$ .

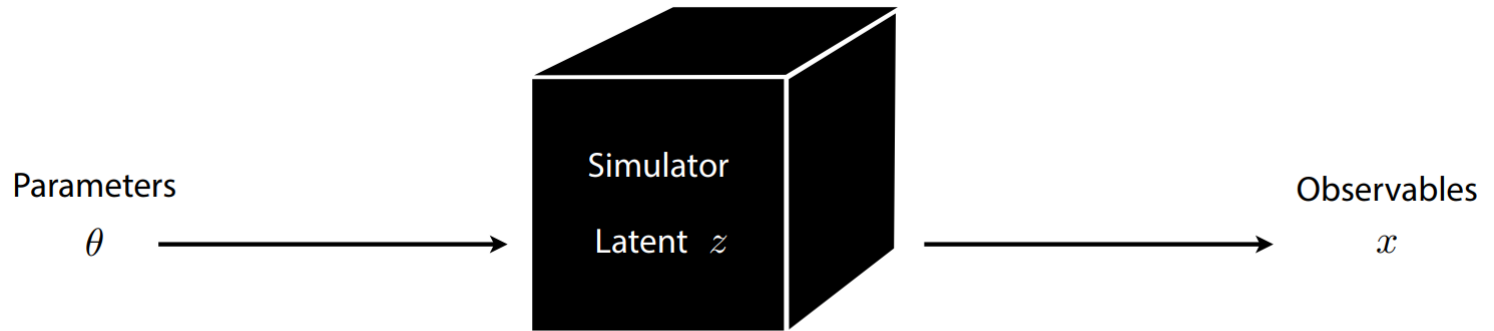
$$p(x|\theta) = \int p(x, z|\theta) dz = \binom{n}{x} \theta^x (1 - \theta)^{n-x}$$

But what if we shift or remove some of the pins?

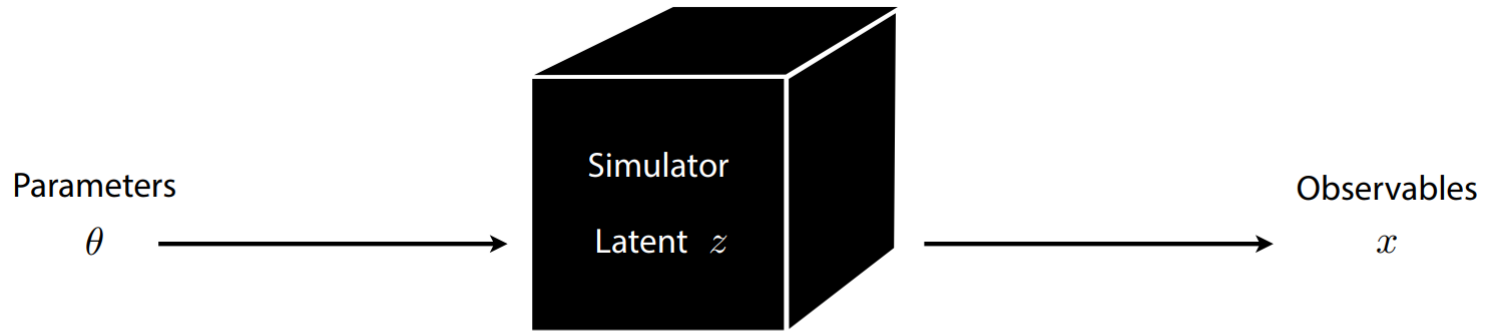
The Galton board is a **metaphore** of simulation-based science:

Galton board device	→	Computer simulation
Parameters $\theta$	→	Model parameters $\theta$
Buckets $x$	→	Observables $x$
Random paths $z$	→	Latent variables $z$ (stochastic execution traces through simulator)

Inference in this context requires **likelihood-free algorithms**.



- Prediction:
- Well-understood mechanistic model
  - Simulator can generate samples



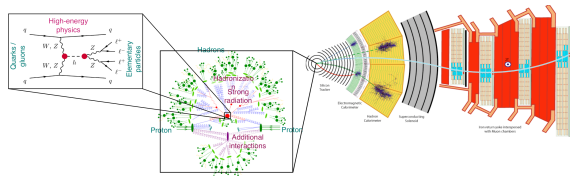
Prediction:

- Well-understood mechanistic model
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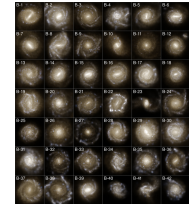
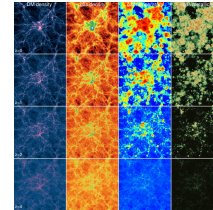
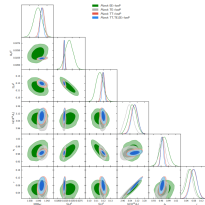
Inference:

- Likelihood function  $p(x|\theta)$  is intractable
- Inference based on estimator  $\hat{p}(x|\theta)$

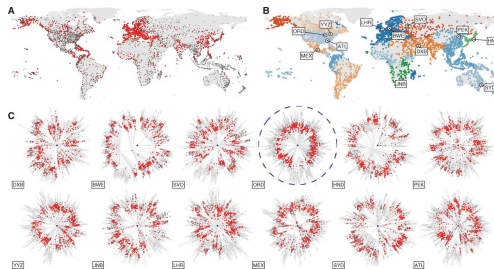
# Applications



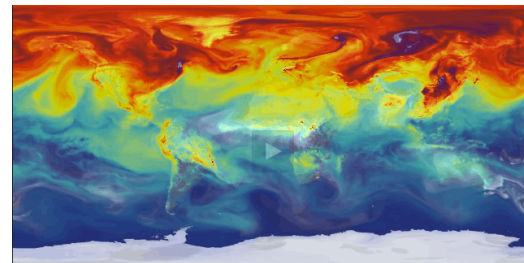
Particle physics



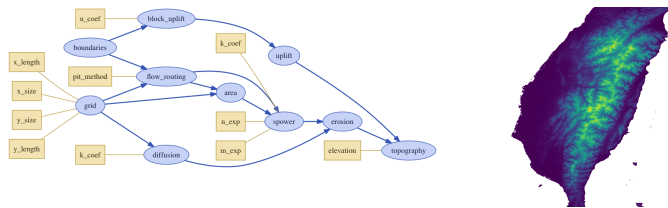
Cosmology



Epidemiology



Climatology

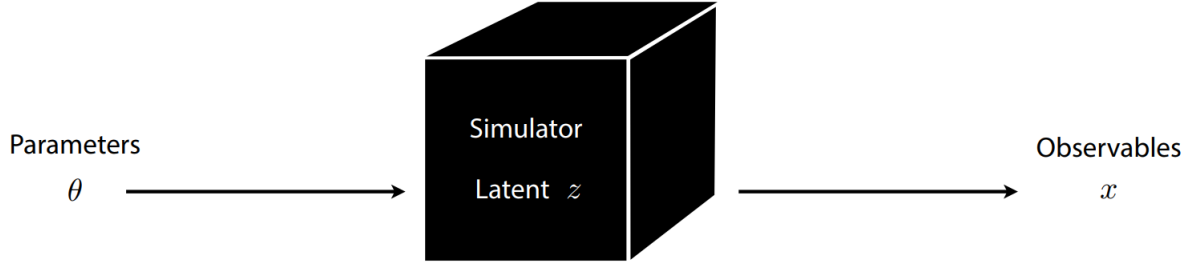


Computational topography

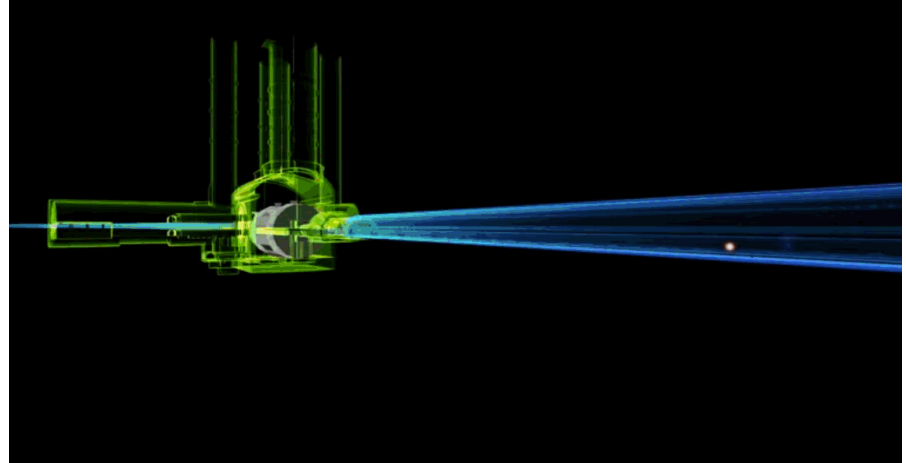


Astronomy

# Particle physics



$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\mu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\mu^a \partial_\nu g_\mu^b g_\mu^c - \frac{1}{4}g^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\mu^e - \partial_\mu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\mu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2\epsilon^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\mu \partial_\nu A_\mu - ig_{cw} (\partial_\mu Z_\mu^0 (W_\mu^+ W_\mu^- - \\
 & W_\mu^+ W_\mu^-) - Z_\mu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+)) + Z_\mu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) - \\
 & ig_{sw} (\partial_\mu A_\mu (W_\mu^+ W_\mu^- - W_\mu^- W_\mu^+) - A_\mu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+)) + A_\mu (W_\mu^+ \partial_\nu W_\mu^- - \\
 & W_\mu^- \partial_\nu W_\mu^+) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\mu^+ W_\mu^- + \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\mu^- W_\mu^+ + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\mu^0 W_\mu^- - \\
 & Z_\mu^0 Z_\mu^0 W_\mu^+ W_\mu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\mu W_\mu^- - A_\mu A_\mu W_\mu^+ W_\mu^-) + g^2 s_w c_w (A_\mu Z_\mu^0 (W_\mu^+ W_\mu^- - \\
 & W_\mu^- W_\mu^+) - 2A_\mu Z_\mu^0 W_\mu^+ W_\mu^-) - \frac{1}{2}\partial_\mu H \partial_\nu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\nu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\nu \phi^0 - \\
 & \beta_h \left( \frac{2M^2}{g^2} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^2}{g^2} \alpha_h - \\
 & g\alpha_h M (H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-) - \\
 & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & gM W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{\epsilon^2} Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{\epsilon^2} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 & M (\frac{1}{\epsilon^2} Z_\mu^0 \partial_\nu \phi^0 + W_\mu^+ \partial_\nu \phi^- + W_\mu^- \partial_\nu \phi^+) - ig \frac{2\epsilon}{\epsilon^2} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) - ig \frac{1-2\epsilon^2}{2\epsilon^2} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{2}g^2 \frac{1}{\epsilon^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \frac{2\epsilon}{\epsilon^2} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{2\epsilon}{\epsilon^2} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{2\epsilon}{\epsilon^2} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig_s \lambda_\mu^2 (g_\mu^+ g_\mu^+ g_\mu^+ g_\mu^+ - e^4 (\gamma \partial + m_e^2) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^2) \nu^\lambda - \bar{u}_1^\lambda (\gamma \partial + \\
 & m_u^2) u_1^\lambda - \bar{d}_1^\lambda (\gamma \partial + m_d^2) d_1^\lambda + ig_{sw} A_\mu (-e^3 \gamma^\mu e^\lambda) + \frac{3}{2}(\bar{u}_1^\lambda \gamma^\mu u_1^\lambda) - \frac{1}{2}(\bar{d}_1^\lambda \gamma^\mu d_1^\lambda)) + \\
 & \frac{ig}{2\epsilon^2} Z_\mu^0 \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_1^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_1^\lambda) + \\
 & (\bar{u}_1^\lambda \gamma^\mu (1 - \frac{4}{3}s_w^2 + \gamma^5) u_1^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}{}_{\lambda k} e^\lambda) + (\bar{u}_1^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda k} d_1^\lambda) \} + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- \{ (\bar{e}^\lambda U^{lep}{}_{\lambda k} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_1^\lambda C_{\lambda k} \gamma^\mu (1 + \gamma^5) u_1^\lambda) \} + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ \{ -m_e^2 (\bar{\nu}^\lambda U^{lep}{}_{\lambda k} (1 - \gamma^5) e^\lambda) + m_\nu^2 (\bar{\nu}^\lambda U^{lep}{}_{\lambda k} (1 + \gamma^5) e^\lambda) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- \{ m_e^2 (\bar{e}^\lambda U^{lep}{}_{\lambda k} (1 + \gamma^5) \nu^\lambda) - m_\nu^2 (\bar{e}^\lambda U^{lep}{}_{\lambda k} (1 - \gamma^5) \nu^\lambda) \} - \frac{g}{2M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2M} m_e^2 \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2M} m_\nu^2 \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_k M_{\lambda k}^R (1 - \gamma_5) \bar{\nu}_k - \\
 & \frac{1}{4} \bar{\nu}_k M_{\lambda k}^L (1 - \gamma_5) \bar{\nu}_k + \frac{ig}{2M\sqrt{2}} \phi^+ \{ -m_u^2 (\bar{u}_1^\lambda C_{\lambda k} (1 - \gamma^5) d_1^\lambda) + m_d^2 (\bar{u}_1^\lambda C_{\lambda k} (1 + \gamma^5) d_1^\lambda) \} + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- \{ m_d^2 (\bar{d}_1^\lambda C_{\lambda k} (1 + \gamma^5) u_1^\lambda) - m_u^2 (\bar{d}_1^\lambda C_{\lambda k} (1 - \gamma^5) u_1^\lambda) \} - \frac{g}{2M} H (\bar{u}_1^\lambda u_1^\lambda) - \\
 & \frac{g}{2M} H (\bar{d}_1^\lambda d_1^\lambda) + \frac{ig}{2M} m_e^2 \phi^0 (\bar{u}_1^\lambda \gamma^5 u_1^\lambda) - \frac{ig}{2M} m_\nu^2 \phi^0 (\bar{d}_1^\lambda \gamma^5 d_1^\lambda) + G^0 \partial^2 G^0 + g_s f^{abc} \partial_\mu G^a G^b G^c + \\
 & X^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) \bar{X}^- + \bar{X}^0 (\partial^2 - \frac{M^2}{\epsilon^2}) X^0 + Y \partial^2 Y + ig_{cw} W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \partial_\mu \bar{X}^+ X^0) + ig_{sw} W_\mu^+ (\partial_\mu Y X^- - \partial_\mu \bar{X}^+ Y) + ig_{cw} W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \partial_\mu \bar{X}^0 X^+) + ig_{sw} W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu Y X^+) + ig_{cw} Z_\mu^0 (\partial_\mu \bar{X}^+ X^- - \\
 & \partial_\mu \bar{X}^- X^+) + ig_{sw} A_\mu (\partial_\mu \bar{X}^+ X^- - \\
 & \partial_\mu \bar{X}^- X^+) - \frac{1}{2}ig M (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{\epsilon^2} \bar{X}^0 X^0 H) + \frac{1-2\epsilon^2}{2\epsilon^2} ig M (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 & \frac{1}{2\epsilon} ig M (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + ig M s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2}ig M (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
 \end{aligned}$$





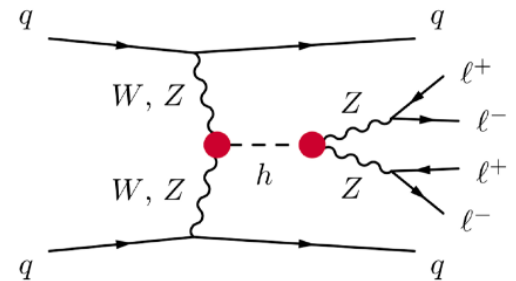
Latent variables

Parameters  
of interest

Parton-level  
momenta

Theory  
parameters

$$z_p \longleftarrow \theta$$



Latent variables

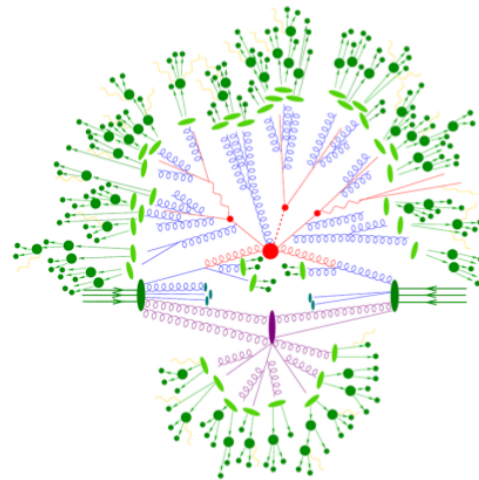
Parameters of interest

Shower splittings

Parton-level momenta

Theory parameters

$z_s$  ←  $z_p$  ←  $\theta$



Latent variables

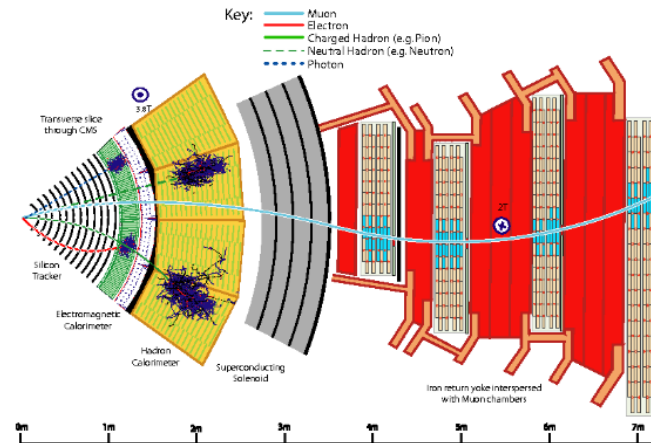
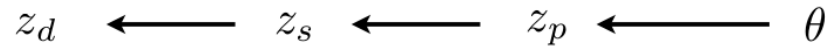
Parameters of interest

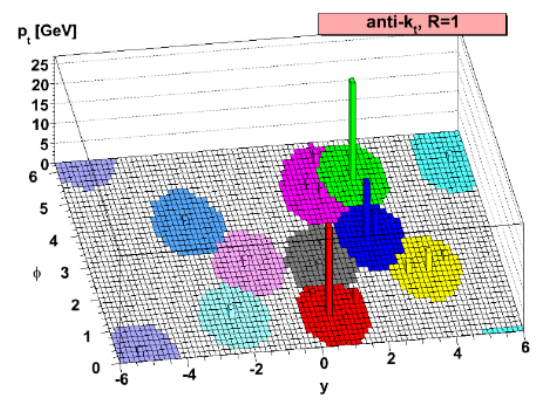
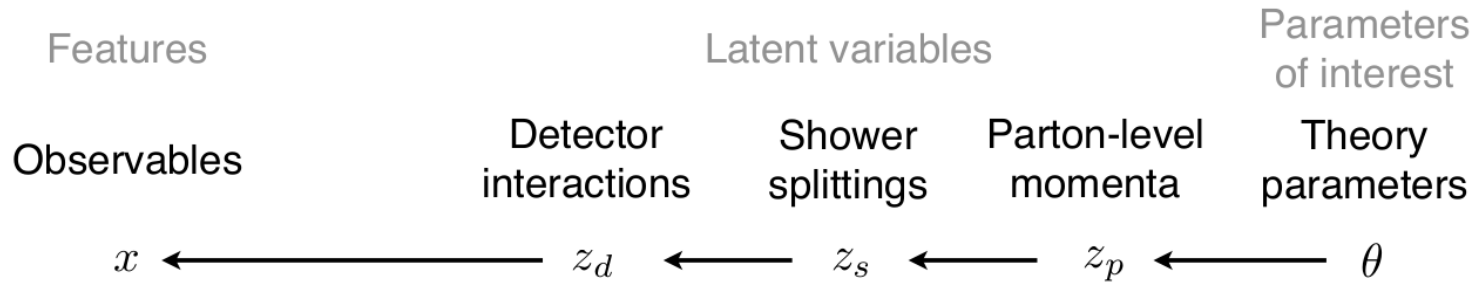
Detector interactions

Shower splittings

Parton-level momenta

Theory parameters





[Image source: M. Cacciari, G. Salam, G. Soyez 0802.1189]

$$p(x|\theta) = \underbrace{\iiint}_{\text{intractable}} p(z_p|\theta)p(z_s|z_p)p(z_d|z_s)p(x|z_d)dz_pdz_sdz_d$$

# Likelihood ratio

The likelihood ratio

$$r(x|\theta_0, \theta_1) = \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

is the quantity that is **central** to many **statistical inference** procedures.

## Examples

- Frequentist hypothesis testing
- Supervised learning
- Bayesian posterior sampling with MCMC
- Bayesian posterior inference through Variational Inference
- Generative adversarial networks
- Empirical Bayes with Adversarial Variational Optimization

The likelihood  $p(x|\theta)$  is actually rarely needed.

*When solving a problem of interest, do not solve a more general problem as an intermediate step. – Vladimir Vapnik*



$$\frac{p_{\mathbf{x}}(\mathbf{x}|\theta_0)}{p_{\mathbf{x}}(\mathbf{x}|\theta_1)} = r(\mathbf{x}; \theta_0, \theta_1)$$

The equation is annotated with two curved arrows: a blue arrow above the fraction pointing from left to right, and a red arrow below the fraction pointing from right to left. The red arrow has a diagonal slash through it, indicating that the reverse direction is not the intended path.

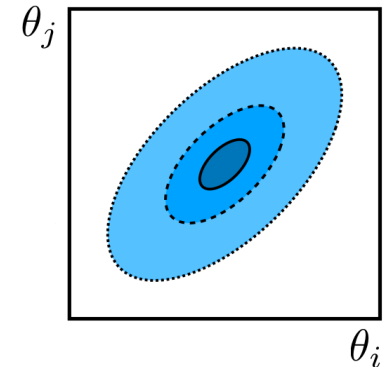
Direct likelihood ratio estimation is simpler than density estimation.  
(This is fortunate, we are in the likelihood-free scenario!)

# The frequentist physicist's way

The Neyman-Pearson lemma states that the likelihood ratio

$$r(x|\theta_0, \theta_1) = \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

is the **most powerful test statistic** to discriminate between a null hypothesis  $\theta_0$  and an alternative  $\theta_1$ .



*IX. On the Problem of the most Efficient Tests of Statistical Hypotheses.*

*By J. NEYMAN, Nencki Institute, Soc. Sci. Lit. Varsoviensis, and Lecturer at the Central College of Agriculture, Warsaw, and E. S. PEARSON, Department of Applied Statistics, University College, London.*

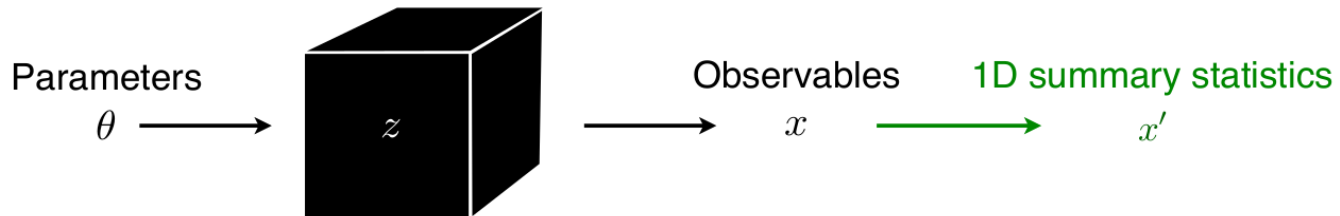
*(Communicated by K. PEARSON, F.R.S.)*

(Received August 31, 1932.—Read November 10, 1932.)

CONTENTS.

	PAGE.
I. Introductory . . . . .	289
II. Outline of General Theory . . . . .	293
III. Simple Hypotheses . . . . .	





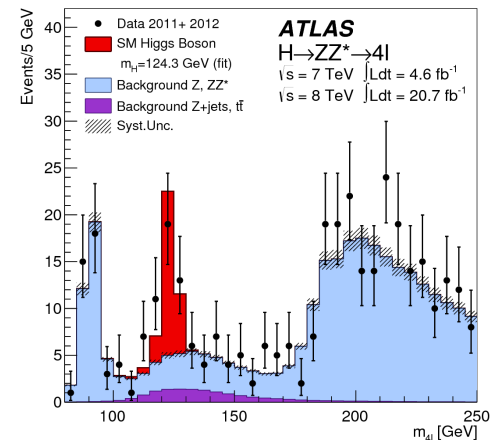
Define a projection function  $s : \mathcal{X} \rightarrow \mathbb{R}$  mapping observables  $x$  to a summary statistics  $x' = s(x)$ .

Then, **approximate** the likelihood  $p(x|\theta)$  as

$$p(x|\theta) \approx \hat{p}(x|\theta) = p(x'|\theta).$$

From this it comes

$$\frac{p(x|\theta_0)}{p(x|\theta_1)} \approx \frac{\hat{p}(x|\theta_0)}{\hat{p}(x|\theta_1)} = \hat{r}(x|\theta_0, \theta_1).$$

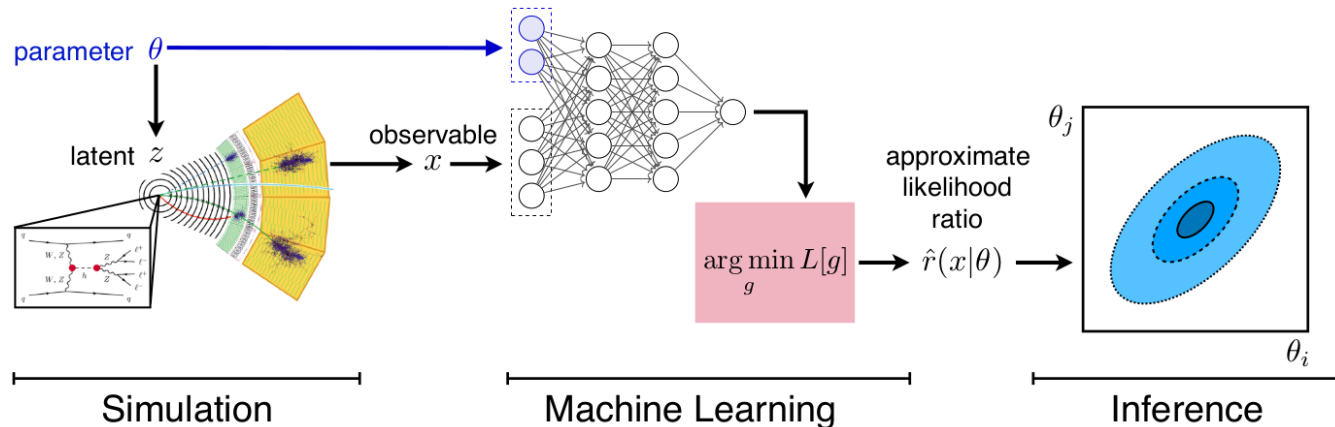


# CARL

Supervised learning provides a way to **automatically** construct  $s$ :

- Let us consider a binary classifier  $\hat{s}$  (e.g., a neural network) trained to distinguish  $x \sim p(x|\theta_0)$  from  $x \sim p(x|\theta_1)$ .
- $\hat{s}$  is trained by minimizing the cross-entropy loss

$$L_{XE}[\hat{s}] = -\mathbb{E}_{p(x|\theta)\pi(\theta)} [1(\theta = \theta_0) \log \hat{s}(x) + 1(\theta = \theta_1) \log(1 - \hat{s}(x))]$$



The solution  $\hat{s}$  found after training approximates the optimal classifier

$$\hat{s}(x) \approx s^*(x) = \frac{p(x|\theta_1)}{p(x|\theta_0) + p(x|\theta_1)}.$$

Therefore,

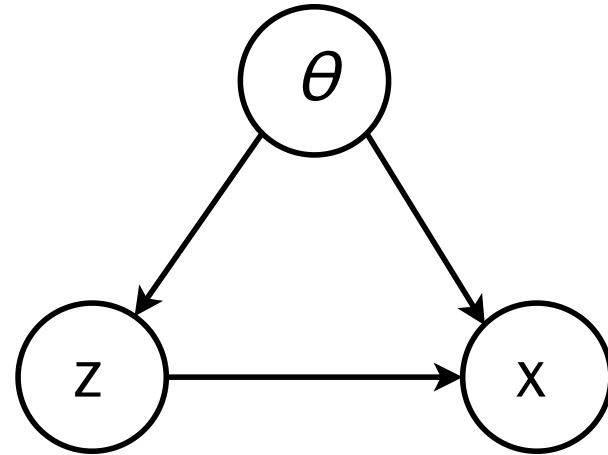
$$r(x|\theta_0, \theta_1) \approx \hat{r}(x|\theta_0, \theta_1) = \frac{1 - \hat{s}(x)}{\hat{s}(x)}$$

That is, supervised classification is equivalent to likelihood ratio estimation.

# Bayesian inference

For a given model  $p(x, z, \theta)$ ,  
Bayesian inference usually consists in  
computing the posterior

$$p(\theta|x) = \frac{p(x|\theta)p(\theta)}{p(x)}.$$



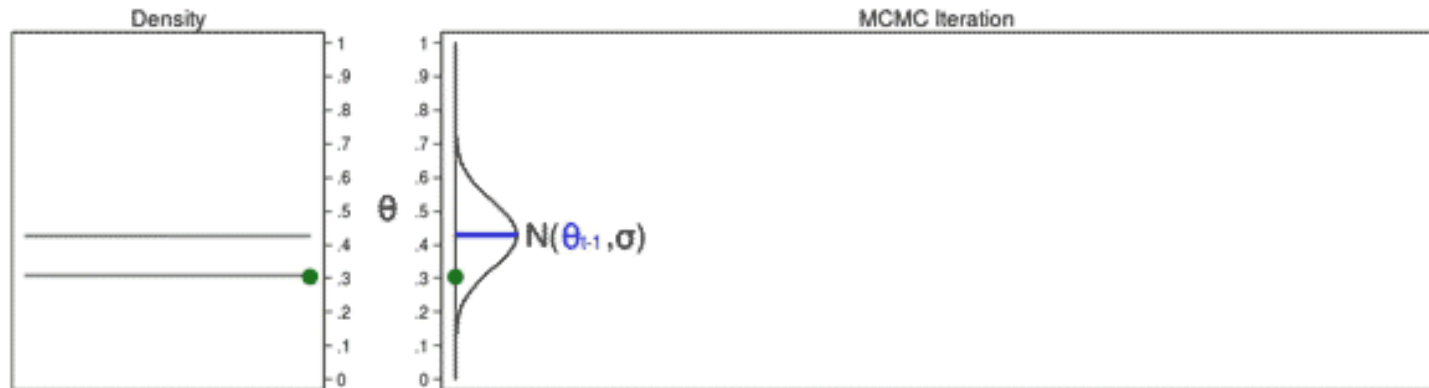
For most cases, this is intractable since it requires evaluating the evidence

$$p(x) = \int p(x|\theta)p(\theta)d\theta.$$

In the likelihood-free scenario, this is even less tractable since we cannot even evaluate the likelihood

$$p(x|\theta) = \int p(x, z|\theta)dz.$$

# Posterior sampling



$$\text{Step 1: } r(\theta_{new}, \theta_{t-1}) = \frac{\text{Posterior}(\theta_{new})}{\text{Posterior}(\theta_{t-1})} = \frac{\text{Beta}(1,1,0.306) \times \text{Binomial}(10,4,0.306)}{\text{Beta}(1,1,0.429) \times \text{Binomial}(10,4,0.429)} = 0.834$$

$$\text{Step 2: Acceptance probability } \alpha(\theta_{new}, \theta_{t-1}) = \min\{r(\theta_{new}, \theta_{t-1}), 1\} = \min\{0.834, 1\} = 0.834$$

$$\text{Step 3: Draw } u \sim \text{Uniform}(0,1) = 0.617$$

$$\text{Step 4: If } u < \alpha(\theta_{new}, \theta_{t-1}) \rightarrow \text{If } 0.617 < 0.834 \quad \text{Then } \theta_t = \theta_{new} = 0.306$$

$$\text{Otherwise } \theta_t = \theta_{t-1} = 0.429$$

# Likelihood-free MCMC with Approximate Likelihood Ratios

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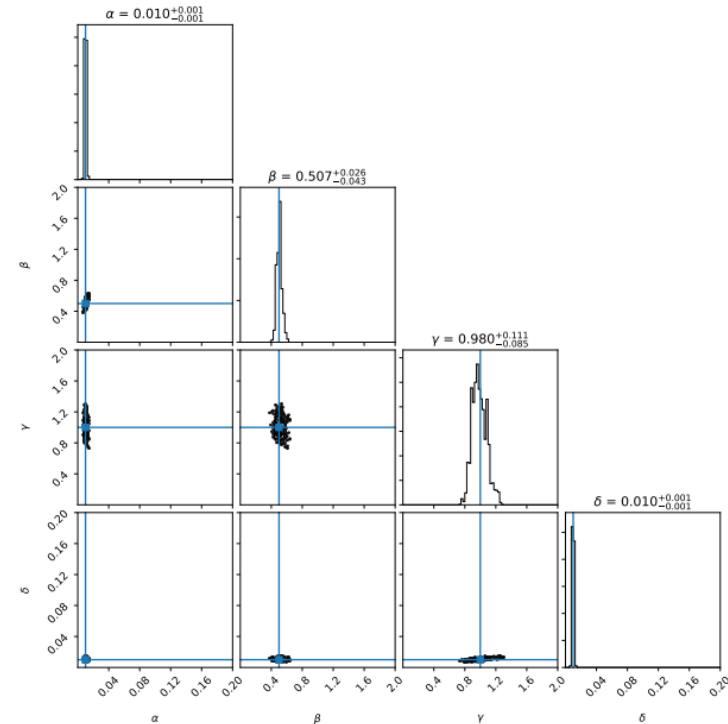
## Algorithm 2 Likelihood-free Metropolis-Hastings

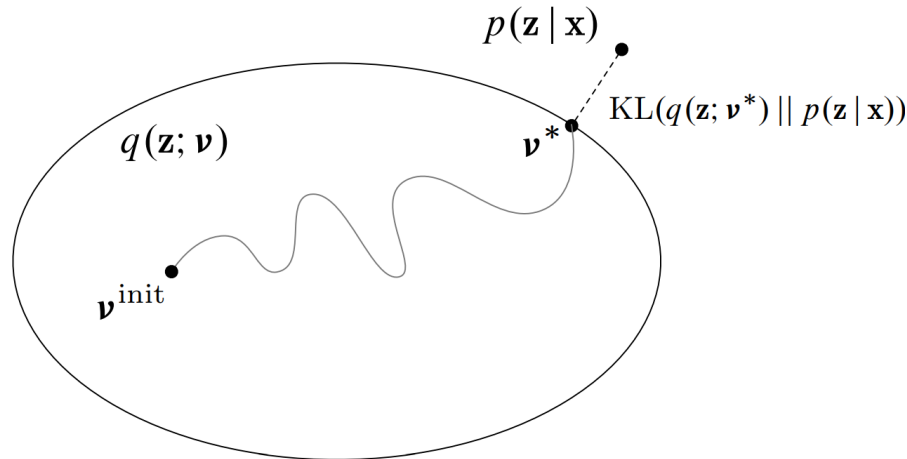
*Inputs:* Initial parameter  $\theta_0$ .  
Transition distribution  $q(\theta)$ .  
Parameterized classifier  $s(\mathbf{x}, \theta)$ .  
Observations  $\mathcal{O}$ .

*Outputs:* Markov chain  $\theta_{0:n}$

*Hyperparameters:* Steps  $n$ .

- 1:  $t \leftarrow 0$
- 2:  $\theta_t \leftarrow \theta_0$
- 3: **for**  $t < n$  **do**
- 4:  $\theta' \sim q(\theta | \theta_t)$
- 5:  $\lambda \leftarrow \sum_{\mathbf{x} \in \mathcal{O}} \log \hat{r}_e(\mathbf{x}, \theta') - \sum_{\mathbf{x} \in \mathcal{O}} \log \hat{r}_e(\mathbf{x}, \theta_t)$
- 6:  $\rho \leftarrow \min \left\{ \exp(\lambda) \frac{q(\theta_t | \theta')}{q(\theta' | \theta_t)}, 1 \right\}$
- 7:  $\theta_{t+1} \leftarrow \begin{cases} \theta' & \text{with probability } \rho \\ \theta_t & \text{with probability } 1 - \rho \end{cases}$
- 8:  $t \leftarrow t + 1$
- 9: **end for**
- 10: **return**  $\theta_{0:n}$





## Likelihood-free Variational inference

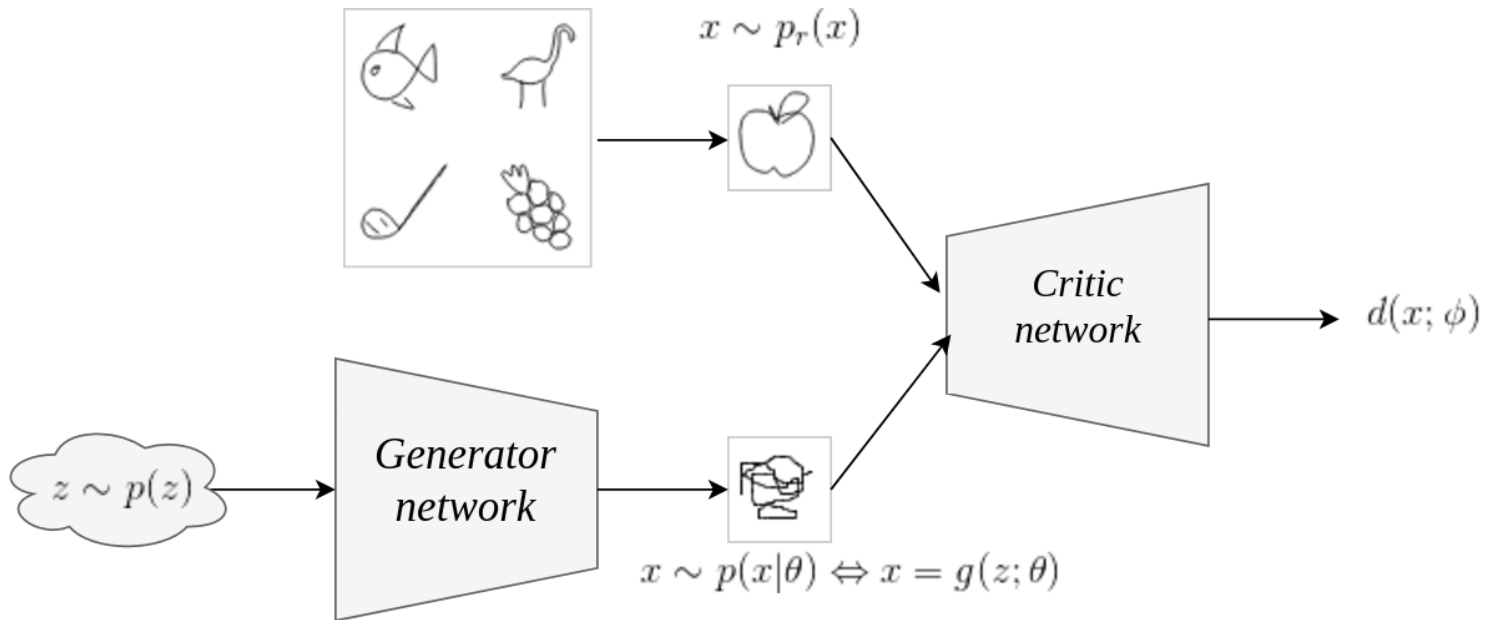
Let  $q(\mathbf{x}_n)$  be the empirical distribution on the observations  $\mathbf{x}$  and consider using it in a “variational joint”  $q(\mathbf{x}_n, \mathbf{z}_n | \beta) = q(\mathbf{x}_n)q(\mathbf{z}_n | \mathbf{x}_n, \beta)$ . Now subtract the log empirical  $\log q(\mathbf{x}_n)$  from the ELBO above. The ELBO reduces to

$$\mathcal{L} \propto \mathbb{E}_{q(\beta)} [\log p(\beta) - \log q(\beta)] + \sum_{n=1}^N \mathbb{E}_{q(\beta)q(\mathbf{z}_n | \mathbf{x}_n, \beta)} \left[ \log \frac{p(\mathbf{x}_n, \mathbf{z}_n | \beta)}{q(\mathbf{x}_n, \mathbf{z}_n | \beta)} \right]. \quad (4)$$

(Here the proportionality symbol means equality up to additive constants.) Thus the ELBO is a function of the ratio of two intractable densities. If we can form an estimator of this ratio, we can proceed with optimizing the ELBO.

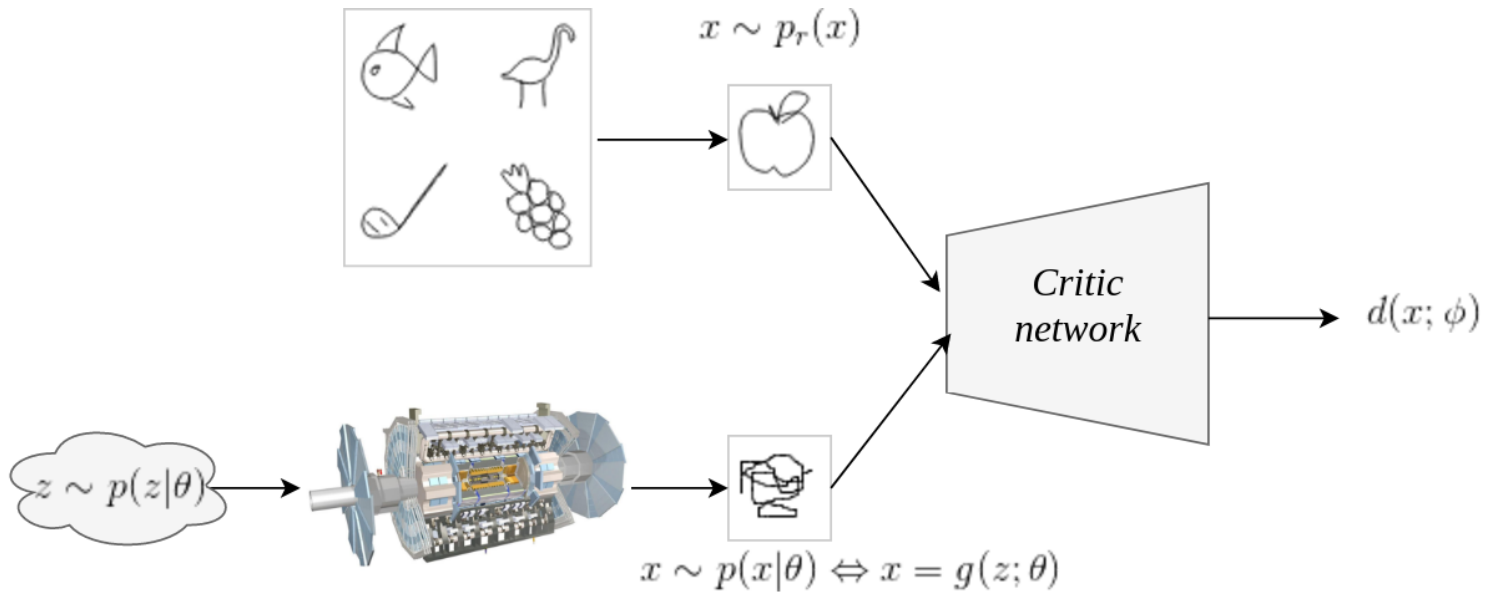
We apply techniques for ratio estimation [51]. It is a key idea in GANs [37, 56], and similar ideas have reappeared in statistics and physics [21, 8].

# Generative adversarial networks





# Adversarial Variational Optimization



Replace  $g$  with an actual scientific simulator!

## Key insights

- Replace the generative network with a non-differentiable forward simulator  $g(z; \theta)$ .
- Let the neural network critic figure out how to adjust the simulator parameters.
- Combine with variational optimization to bypass the non-differentiability by optimizing upper bounds of the adversarial objectives

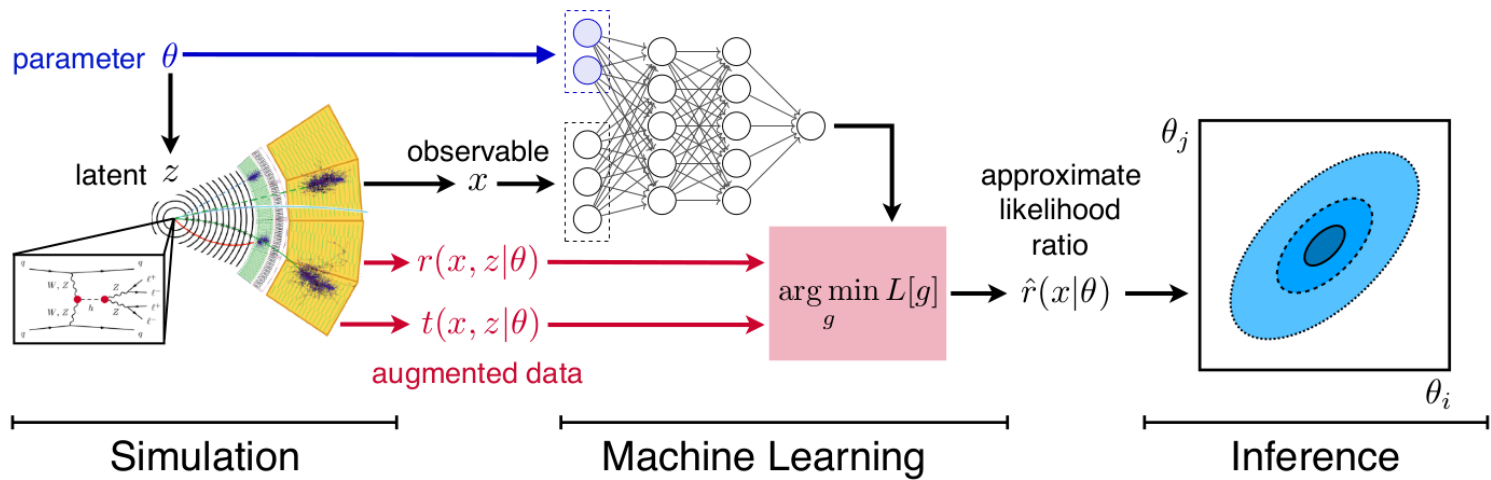
$$U_d(\phi) = \mathbb{E}_{\theta \sim q(\theta; \psi)} [\mathcal{L}_d(\phi)]$$

$$U_g(\psi) = \mathbb{E}_{\theta \sim q(\theta; \psi)} [\mathcal{L}_g(\theta)]$$

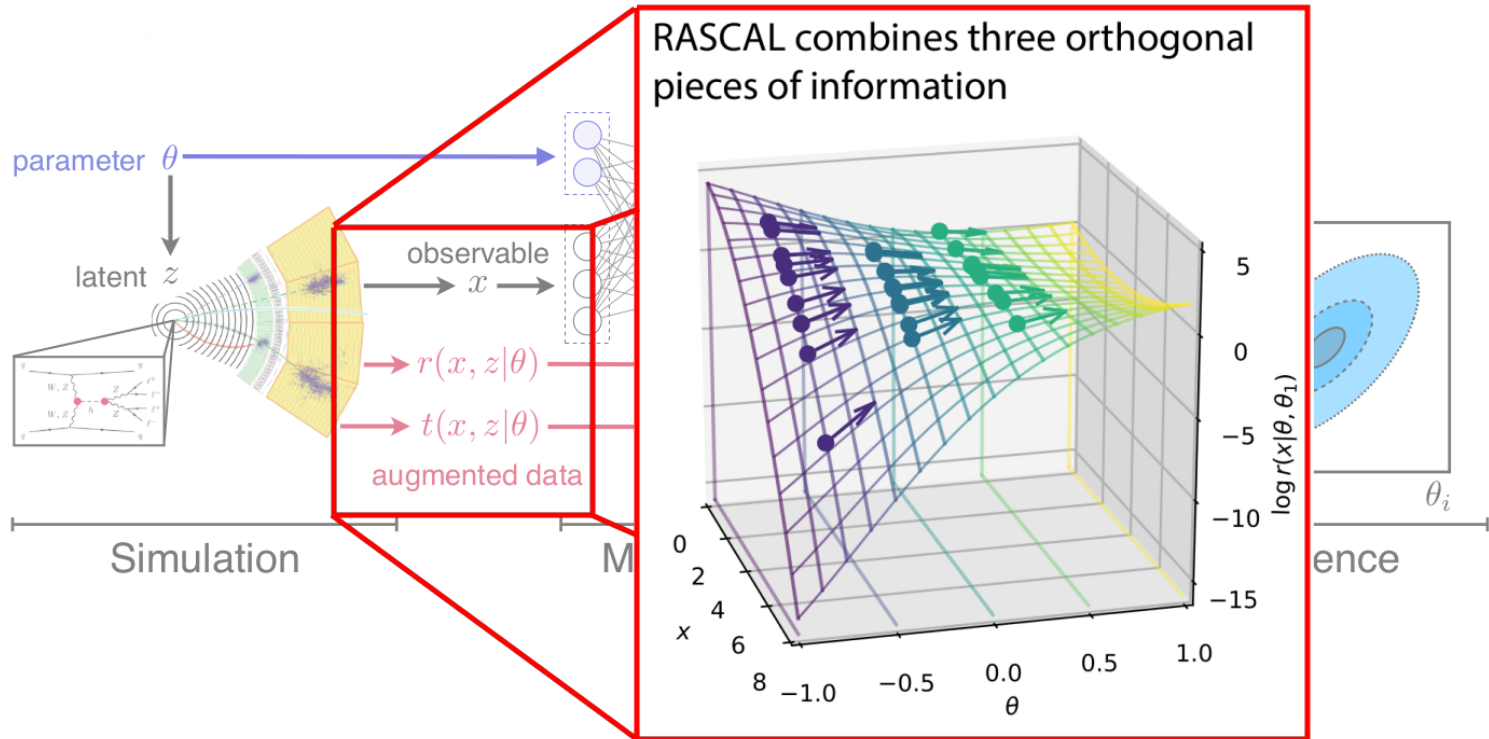
respectively over  $\phi$  and  $\psi$ .

- Effectively, this amounts to empirical Bayes guided by the likelihood ratios estimated from the critic.

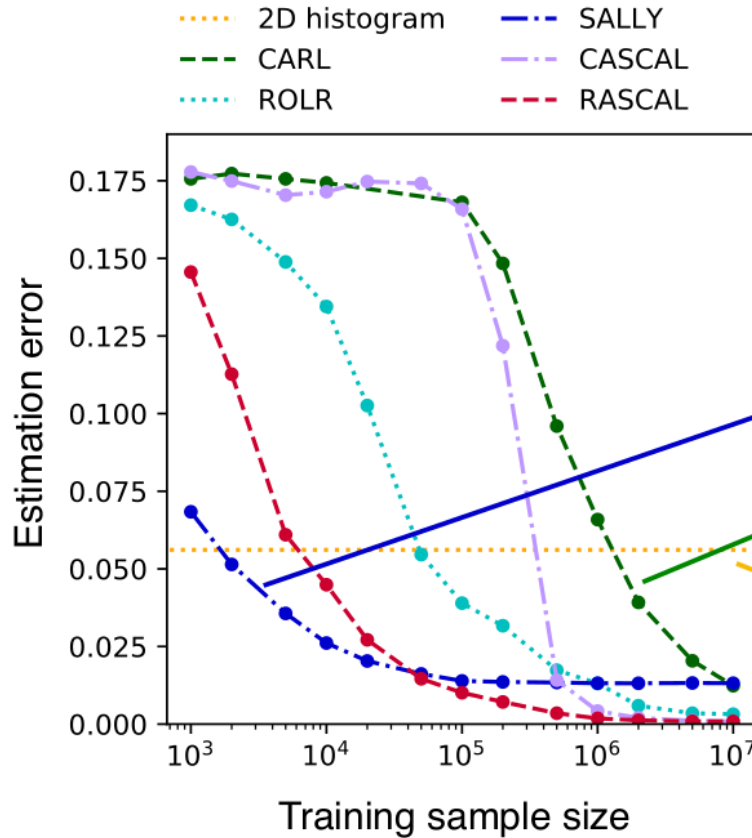
# Mining gold



# Mining gold



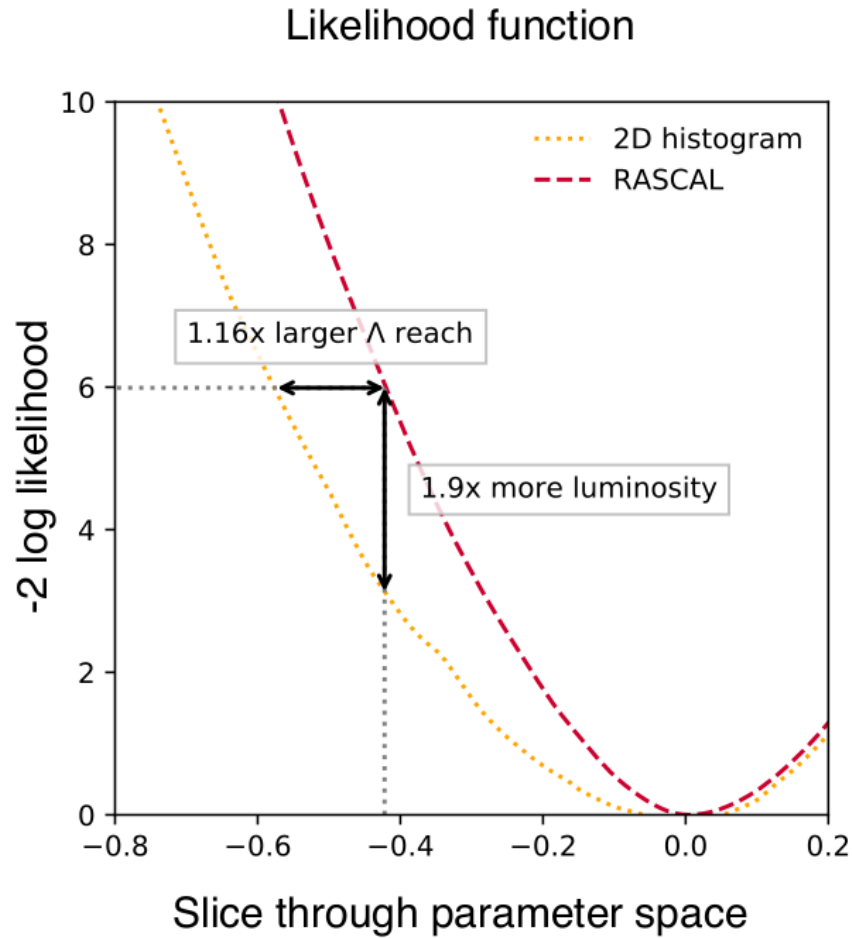
# Increased data efficiency



New techniques  
require less data than  
generic ML method  
[CARL, see K. Cranmer, G. Louppe,  
J. Pavez 1506.02169]

Traditional Histogram

# Better sensitivity



36 events, assuming SM

# Summary

- Much of modern science is based on "likelihood-free" simulations.
- The likelihood-ratio is central to many statistical inference procedures.
- Supervised learning enables likelihood-ratio estimation.
- Better likelihood-ratio estimates can be achieved by mining simulators.

# Collaborators



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The end.

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